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RESEARCH MEMORANDUM

ANALYTICAL INVESTIGATION OF THE USE OF REGENERATION IN COMPRESSOR-TURBINE-PROPELLER SYSTEMS

By

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SUMMARY

The use of regenerative heat exchangers in compressor-turbine-propeller aircraft power plants has been investigated. The effects of regeneration on specific fuel consumption and thrust power have been calculated for various values of the important parameters: altitude, blower compression ratio, maximum turbine-inlet temperature, and heat-exchanger cross-sectional area. The effects of regeneration on fuel requirements and cargo capacity for flights of various durations have been determined.

The investigation showed that reductions in specific fuel consumption could be obtained by means of regenerative heat exchangers. For the same heat-exchanger weight, the reduction in specific fuel consumption would be greater the lower the altitude, the higher the maximum temperature at the turbine inlet, the lower the temperature rise due to compression, and consequently the lower the pressure ratio across the compressor. For flights of sufficient duration, the reduction in fuel weight would be greater than the heat-exchanger weight required to give that reduction. The difference between the reduction in fuel weight and the heat-exchanger weight would indicate that additional cargo could be carried without a change in take-off gross weight when regeneration is used.

As an example, a regenerative heat exchanger that weighs 4000 pounds can reduce by 19 percent, or 650 pounds per hour, the rate of fuel consumption of a power plant operating at sea level with a compression ratio of 6, a capacity of 50 pounds of air per second, a maximum temperature of 1800° F, and a thrust power of 5400 horsepower. For flights of greater duration than $6\frac{1}{2}$ hours, the reduction in fuel weight is greater than the heat-exchanger weight and, for the same take-off weight, additional cargo

can be loaded. For a 16-hour flight, the required fuel weight is reduced by 10,500 pounds and the cargo weight is increased by 6500 pounds.

INTRODUCTION

Regeneration is a well-known means of modifying the thermodynamic cycle of an engine to improve fuel economy. Because regeneration is accomplished by means of heat exchangers, it entails an increase in the weight of a power plant. The use of regeneration for increasing efficiency is common practice for stationary steam-turbine power plants, in which weight is not so important a consideration as in aircraft power plants. The use of regeneration in gas-turbine marine power plants has been considered in references 1 and 2. The National Advisory Committee for Aeronautics has published analyses of regenerative aircraft power plants - reference 3 deals with the regenerative propeller-compressor-engine-turbine-jet system and reference 4 considers, among others, the regenerative compressor-turbine system. The purpose of the present paper is to show in more detail the possibilities and the limitations of the use of regeneration as a means of effecting greater fuel economy and of reducing the net weight of the power plant and fuel of compressor-turbine-propeller aircraft engines. Curves are therefore given that show, for a variety of conditions, the variation with heat-exchanger weight of specific fuel consumption, thrust power, fuel weight, and net change in the take-off weight of power plant and fuel.

DESCRIPTION OF SYSTEM AND METHOD

The compressor-turbine system without regeneration operates on the thermodynamic cycle shown in figure 1. The working fluid undergoes the following processes (fig. 1):

0 to 1	induction from free stream into airplane
1 to 2	mechanical compression
2 to 4	combustion
4 to 5	expansion in turbine
5 to 7	expansion in exhaust nozzle

Regeneration may be effected in the compressor-turbine system by adding a heat exchanger as shown in figure 2.

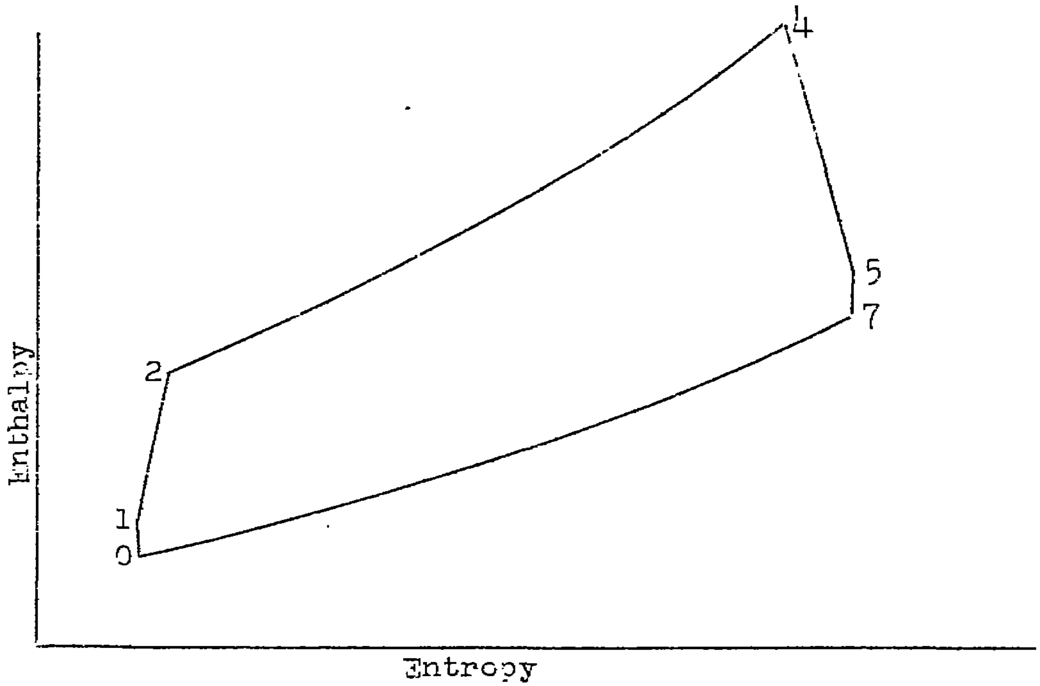

Figure 1.- Nonregenerative compressor-turbine engine cycle.

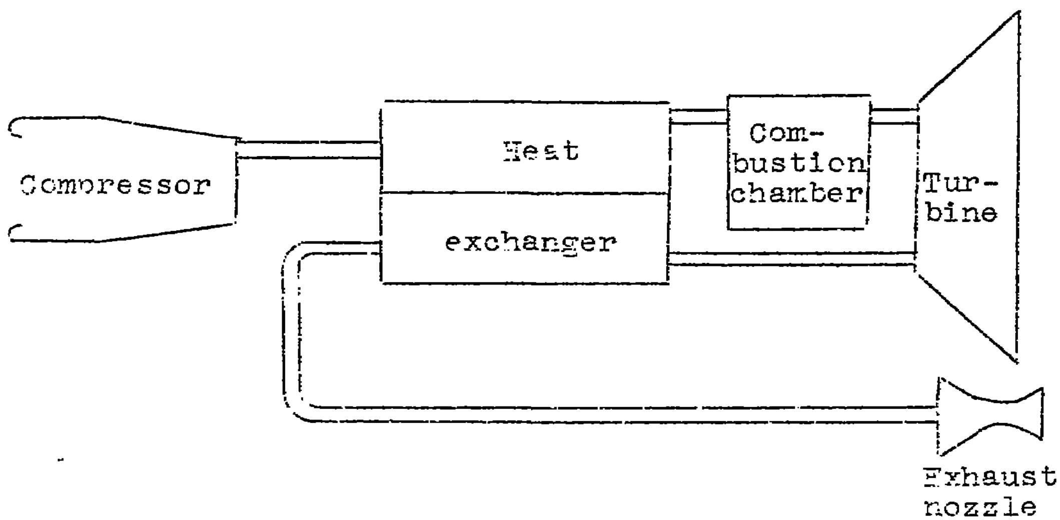

Figure 2.- Regenerative power plant.

Heat is extracted from the fluid between the turbine and the exhaust nozzle and is added to the colder fluid between the compressor and the combustion chamber. The cycle then becomes that shown in figure 3, in which the curve from stations 2 to 3 represents heating in the heat exchanger and the curve from stations 5 to 6 represents cooling in the heat exchanger.

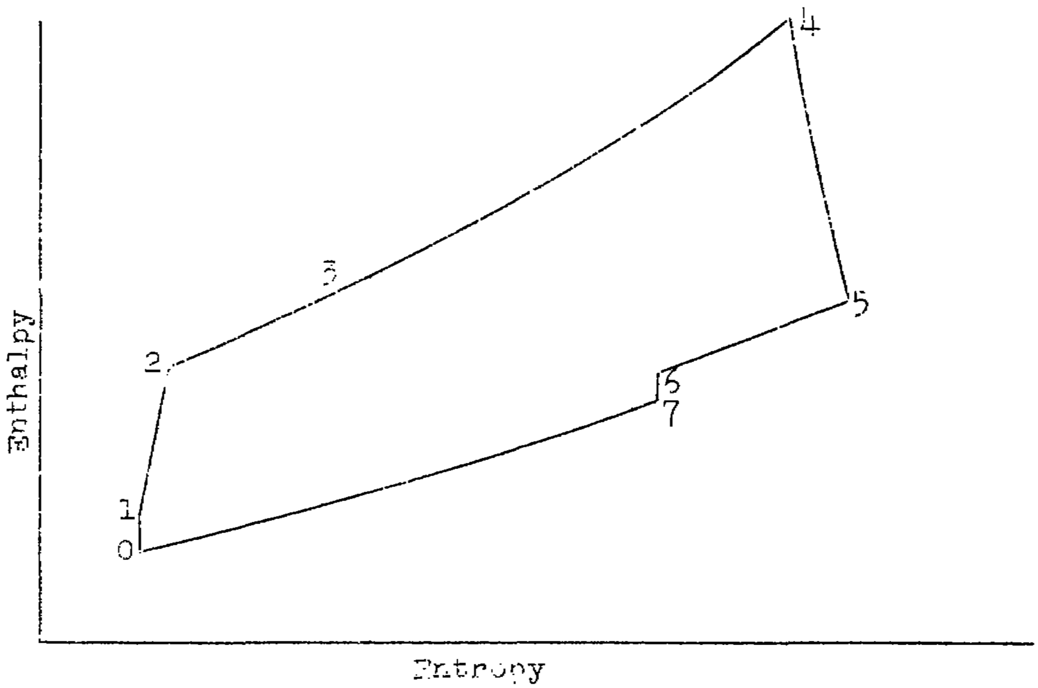

Figure 3.- Regenerative compressor-turbine engine cycle.

The performance of the regenerative power plant is dependent upon a number of parameters, which include altitude, blower compression ratio and efficiency, maximum turbine-inlet temperature, turbine efficiency, heat-exchanger effectiveness, and heat-exchanger cross-sectional area. In this investigation the performance characteristics of the compressor-turbine system were obtained by assigning values to these various parameters (compressor, turbine, and propeller efficiencies of 85 percent were used) and then calculating the change in the condition, or state, of the air between each two adjacent stations shown in figure 3. This procedure, when repeated for a series of values of the various parameters, indicates the performance of the system with regard to specific fuel consumption and power output.

The weights of the heat exchangers that are required to effect the heat transfer necessary for various amounts of regeneration are calculated from a straightforward analysis of the heat-exchange system. The heat exchangers are of the counterflow tube-and-shell type with cores made of bundles of stainless-steel tubes having a 0.020-inch wall thickness and 0.25-inch inside diameter. The tube spacing from center to center is 0.34 inch.

The symbols used in the present paper are given in appendix A. A detailed analysis of the problem, the computational procedure used, a description of the heat exchangers considered, and a discussion of the effect of altitude are presented in appendix B. A discussion of the source of the thermodynamic data for air is presented in appendix C.

RESULTS AND DISCUSSION

Operation at Sea Level

Results for compressor-turbine-propeller systems operating at sea level at an airplane speed of 200 miles per hour are presented in figures 4 to 7. The reduction in specific fuel consumption that can be obtained by means of regeneration is given in figure 4. For this plot the abscissa is the weight of the heat-exchanger installation (core, shell, and ducting) per unit rate of air flow, the pressure ratio across the compressor is 6, and the results are for maximum fluid temperatures (temperature at entrance to the turbine) of 1500° F and 1800° F.

With a maximum allowable temperature at the turbine inlet of 1800° F, the specific fuel consumption of the system is 0.63 pound of fuel per horsepower-hour without regeneration. When a heat exchanger that weighs 50 pounds per pound of air per second and that has an open cross-sectional area given by $A/W = 0.1$ square foot per pound per second is added to the system, the specific fuel consumption drops to 0.50 pound of fuel per horsepower-hour, a decrease of 21 percent (fig. 4). The relatively small gains that result when the weight is increased from 50 to 150 pounds per pound of air per second illustrate the law of diminishing returns, which holds for heat-exchange systems. The reduction in specific fuel consumption obtained with a given heat-exchanger weight is somewhat less for a maximum temperature of 1500° F than for 1800° F.

At low values of regenerator weight, the heat exchangers with the larger cross-sectional areas cause smaller reductions in specific fuel consumption than those with the smaller cross-sectional areas (fig. 4). These reductions for the larger cross-sectional areas are brought about because, as the cross-sectional area is increased, the velocity and the Reynolds number of the flow in the heat-exchanger tubes, and therefore the heat-transfer coefficient, decrease and consequently the rate of heat transfer decreases.

The use of regeneration decreases the power output of a system but the decrease may be inappreciable. There are two reasons for the decrease in power output caused by regeneration. One is that the available energy in the working fluid at the entrance to the exhaust nozzle is decreased by the loss of heat in the regenerator. For the propeller-driven system the reduction in power output from this cause is very small. The other reason for the reduction in power output caused by regeneration is the pressure loss resulting from friction in the heat-exchanger tubes. This pressure loss is a function of the dynamic pressure of the flow in the tubes and is therefore dependent upon the value of the parameter A/W. The effect of A/W is shown in figure 5, which gives the thrust power developed by the system as a function of heat-exchanger weight for several values of A/W. For heat exchangers of relatively large frontal area ($A/W = 0.2$ sq ft/lb/sec), the pressure drop is quite small and has no appreciable effect on the output. For heat exchangers of relatively small frontal area ($A/W = 0.05$ sq ft/lb/sec), the thrust power developed by the system decreases from an original value of 108 horsepower per pound per second to a value of 101 horsepower per pound per second for a heat-exchanger weight of 50 pounds per pound of air per second.

The fact that reductions in specific fuel consumption can be obtained by use of regeneration (fig. 4) means that the weight of the fuel required for a flight of given duration can be reduced by the addition of a heat exchanger. The change in fuel weight as a function of heat-exchanger weight for flights of various durations is given in figure 6. Considerable fuel savings are possible for flights of long duration. As an example, for a 16-hour flight the use of a regenerative heat exchanger that weighs 80 pounds per pound of air per second can reduce fuel consumption by 210 pounds per pound of air per second (fig. 6(a)). If the propulsive system uses 50 pounds of air per second (and accordingly develops a thrust power of 5400 hp, fig. 5(a)), the saving is 10,500 pounds of fuel for a 16-hour flight.

The fact that regenerators can reduce the specific fuel consumption also means that, for flights of sufficient duration, regenerators can reduce the net weight of the power plant and fuel at the beginning of a flight. The net change in the initial weight of the power plant and fuel as a function of the heat-exchanger weight for flights of various durations is given in figure 7. For flights of 4 hours or less, regeneration in the system with a compression ratio of 6 leads to no reduction in the net weight of the power plant and fuel. For flights longer than 4 hours, a reduction in the net weight of the power plant and fuel is obtained. For a flight of 16 hours, as an example, the use of a heat exchanger that weighs 80 pounds per pound of air per second gives a net weight reduction of 130 pounds per pound of air per second. If the rate of air consumption of the power plant is 50 pounds per second, the net weight reduction is 6500 pounds. This net reduction is effected by the use of a 4000-pound heat exchanger that reduces the required fuel weight by 10,500 pounds.

Figure 7 is plotted for $A/W = 0.2$ square foot per pound per second. For this value of A/W, the use of regenerators causes no reduction in the power output of the system (fig. 5). In the present paper only the changes in the fuel weight and the net weight of the power plant and fuel that are effected at constant thrust power are shown. In other words, all plots of change in fuel weight and of net weight change herein are based on heat-exchanger cross-sectional areas chosen in such a way that the heat exchangers have no effect on the thrust-power output. If, therefore, the weight of the cargo is increased by the amount that the net weight of the power plant and fuel is decreased, the gross weight, the wing loading, and the power loading of the airplane at take-off remain the same with regeneration as without regeneration. It has been assumed that no changes in the structural weight of the airplane result from the use of regeneration if the total weight of the power plant, fuel, and cargo remains constant.

It should be pointed out that greater reductions in fuel weight than those shown herein are possible if the heat-exchanger cross-sectional areas are chosen in such a way that the reductions in specific fuel consumption are greater than those obtained with constant thrust power and that, therefore, the thrust power is reduced. These reductions in fuel weight can be obtained from the curves given herein. The present investigation has not been extended to include the derivation of the effects on the net weight of the power plant and fuel of the use of cross-sectional areas that cause losses in thrust power. That the reductions in net weight may possibly be greater than those shown herein should be mentioned.

The preceding discussion has been concerned principally with a compressor-turbine-propeller system in which the pressure ratio across the compressor was 6. A rather detailed discussion has been given in order to explain the effects of certain parameters and to show the method of presentation used herein. The effects of regeneration when the compression ratio is other than 6 should also be shown. The performance of a nonregenerative compressor-turbine-propeller system at sea level is presented in figures 8 and 9. The data for these plots were calculated by the same method used in analyzing the regenerative system, with the exception that the heat-exchanger weight and pressure drop were taken equal to zero.

As the compression ratio is varied between 2 and 9 (fig. 8), the power output goes through a maximum at a compression ratio of approximately 6. As the compression ratio is increased from 2, the specific fuel consumption decreases toward a minimum, which it has nearly reached at a compression ratio of 6 (fig. 9). The effects of regeneration have therefore been investigated for the range of compression ratio from 2 to 6. The results are presented in figures 10 to 21, which show the variation with regenerator weight of specific fuel consumption, thrust power, fuel weight change, and net weight change. The reductions in specific fuel consumption, fuel weight, and the net weight of the power plant and fuel are greater for a compression ratio of 4 (figs. 10 to 13) than for a compression ratio of 6. For a compression ratio of 3 (figs. 14 to 17), the reductions are larger than those for a compression ratio of 4. For a compression ratio of 2 (figs. 18 to 21), very large reductions in specific fuel consumption (approx. 50 percent) and in fuel weight are shown. For flights of long duration, very large reductions in take-off weight of power plant and fuel are shown. Regeneration is therefore a means of effecting a large improvement in the economy of a system that, on account of a low compression ratio, is very uneconomical without regeneration.

The regenerative system with a compression ratio of 2 should be compared, however, with nonregenerative systems with compression ratios of 4 and 6. In order for the regenerative system with a compression ratio of 2 to have the same specific fuel consumption as the nonregenerative system with a compression ratio of 4 (0.74 lb/hp-hr, fig. 9), a heat-exchanger weight of 130 pounds per pound of air per second is needed (fig. 18). In order for the regenerative system with a compression ratio of 2 to have the same specific fuel consumption as the nonregenerative system with a compression ratio of 6 (0.63 lb/hp-hr, fig. 9), a heat-exchanger weight of more than 200 pounds per pound of air per second is required (fig. 18). The use of additional stages of compression in the system with a compression ratio of 2 can give the same reduction in specific fuel consumption that the use of regeneration can give and with much less increase in weight. Furthermore, with a compression ratio of 2, the power output is only 64 horsepower per pound of air per second (fig. 8) but, with a compression ratio of 4, the power output is 99 horsepower per pound of air per second and, with a compression ratio of 6, the power output is 109 horsepower per pound of air per second. The nonregenerative power plant with a compression ratio of 4 to 6 therefore weighs less and develops more power than the regenerative power plant of the same specific fuel consumption but with a compression ratio of 2.

Operation at 25,000 Feet

The effects of regeneration for compressor-turbine-propeller systems operating at an altitude of 25,000 feet and at an airplane speed of 300 miles per hour are shown in figures 22 to 33. Results for a system with a compression ratio of 4 are given in figures 22 to 25, with a compression ratio of 3 in figures 26 to 29, and with a compression ratio of 2 in figures 30 to 33. These figures show that specific fuel consumption and fuel weight can be reduced by means of regeneration and that, for flights of sufficient duration, the net weight of the power plant and fuel can be reduced.

Comparison of the performance at an altitude of 25,000 feet (figs. 22 to 33) with that at sea level (figs. 10 to 21) shows that the reductions made possible by the use of regeneration are less for operation at 25,000 feet than for operation at sea level. The better performance at sea level is due to the difference in the density of the atmosphere at the two altitudes. The relation between heat-exchanger weight and air density (appendix B), when other conditions (pressure drop and dynamic pressure in the heat exchanger and heat-exchanger effectiveness) are held constant, is

$$\frac{\text{Weight}}{W} \propto \left(\frac{1}{\text{Density}}\right)^{0.4}$$

and the relation between heat-exchanger cross-sectional area and air density is

$$\frac{A}{W} \propto \left(\frac{1}{\text{Density}}\right)^{0.5}$$

As an example, at sea level and with $A/W = 0.2$, a heat-exchanger weight of 50 pounds per pound of air per second gives about a 20-percent reduction in specific fuel consumption for a compression ratio of 3 (fig. 14). At 25,000 feet, where the density is substantially one-half that at sea level, A/W must have the value 0.3 for a 20-percent reduction in specific fuel consumption, and the weight of the heat exchanger is 65 pounds per pound of air per second (fig. 26). The fact that at the higher altitudes more heat-exchanger weight is required per pound of air per second than at the lower altitudes means that, for the same take-off weight, regeneration is less effective at the higher than at the lower altitudes as a means of increasing cargo capacity. Comparison of the figures that show the change in fuel weight and in net weight for operation at 25,000 feet with the corresponding figures for operation at sea level shows, however, that the differences in the changes in fuel and net weight are not large.

Operation at 40,000 Feet

At 40,000 feet, where the density is substantially one-fourth that at sea level, heat exchangers must be $1\frac{3}{4}$ times as heavy per unit rate of air flow as at sea level and, for the same percentage reduction in specific fuel consumption, must have twice the cross-sectional area per unit rate of air flow as at sea level. The weight reduction caused by the use of regeneration would therefore be less than that at sea level. The effects of regeneration for the power plant having a compression ratio of 3 and operating at an altitude of 40,000 feet at 450 miles per hour are presented in figures 34 to 37. The reductions at 40,000 feet in fuel weight and in net weight of power plant and fuel are less than the reductions at sea level for power plants of the same compression ratio and flights of the same duration. The reductions are nevertheless large for flights of long duration.

CONCLUSIONS

The following conclusions have been drawn with regard to the application of regeneration to compressor-turbine-propeller aircraft power plants:

1. Reductions in the specific fuel consumption would be obtained by means of regenerative heat exchangers.

2. For the same heat-exchanger weight, the reduction in specific fuel consumption would be greater the lower the altitude, the higher the maximum temperature at the turbine inlet, the lower the temperature rise due to compression, and consequently the lower the pressure ratio across the compressor.

3. For a system operating at sea level with a compression ratio of 6 (the compression ratio that gave approximately minimum specific fuel consumption and maximum power in the nonregenerative system at sea level),

reductions in specific fuel consumption of approximately 20 percent would be obtained without a reduction in thrust power and by use of a regenerator that weighed 50 pounds per pound of air per second.

4. For a system operating at sea level with a very low compression ratio of 2 (a compression ratio that results in a very high specific fuel consumption in the nonregenerative system), reductions in specific fuel consumption of approximately 50 percent would be obtained by means of regeneration without a reduction in thrust power. The use of additional stages of compression, however, would give the same reduction in specific fuel consumption as the use of regeneration, would increase the power-plant weight less than the use of regeneration, and would also increase the thrust power.

5. For flights of sufficient duration, the reduction in fuel weight would be greater than the heat-exchanger weight required to give that reduction. The difference between the reduction in fuel weight and the heat-exchanger weight would indicate that additional cargo could be carried without a change in take-off gross weight when regeneration is used.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., November 26, 1945

APPENDIX A

SYMBOLS

A	open frontal area of heat exchanger, sq ft
c_p	specific heat of fluid at constant pressure, Btu/lb/°F
c_v	specific heat of fluid at constant volume, Btu/lb/°F
D	diameter of heat-exchanger tubes, ft
E	enthalpy of fluid, Btu/lb
ΔE_i	difference between inlet enthalpies of hot and cold fluids, Btu/lb
g	acceleration due to gravity, ft/sec/sec
h	coefficient of heat transfer, Btu/sec/sq ft/°F
H	total, or stagnation, pressure, lb/sq ft
J	mechanical equivalent of heat (778 ft-lb/Btu)
L	length of heat-exchanger tubes, ft
N	number of heat-exchanger tubes
p	static pressure, lb/sq ft
Δp	static-pressure drop due to friction, lb/sq ft
P	thrust power, hp
q	dynamic pressure, lb/sq ft
Q	rate of heat transfer, Btu/sec
R	Reynolds number; also, gas constant where specified
R_c	compression ratio of compressor
S	surface area for heat transfer, sq ft

T temperature, °F

T_{max} maximum (turbine-inlet) temperature, °F

U over-all coefficient of heat transfer between two fluids, Btu/sec/sq ft/°F

V speed, ft/sec

W weight rate of flow of air, lb/sec

W_f weight rate of flow of fuel, lb/sec

γ ratio of specific heats (c_p/c_v)

ζ mean temperature difference between hot fluid and cold fluid divided by inlet-temperature difference

η rise in temperature of cold fluid divided by inlet-temperature difference

η_c efficiency of compressor

η_T efficiency of turbine

μ coefficient of viscosity of fluid, slug/ft-sec

ξ drop in temperature of hot fluid divided by inlet-temperature difference

ρ mass density of fluid, slug/cu ft

Subscripts:

1, 2 fluids in heat exchanger

0 to 53 stations

o free stream

APPENDIX B

ANALYSIS AND COMPUTATIONAL PROCEDURE

Regenerative Cycle

A qualitative representation of the changes in the condition of the working fluid as it passes through the various parts of the regenerative cycle is given in figure 38. The various parts of the cycle are

0 to 10	induction into airplane and diffusion
10 to 21	compression by blower
21 to 22	addition of energy in heat exchanger
22 to 30	addition of energy by combustion
30 to 41	operation of turbine
41 to 43	subtraction of energy in heat exchanger
43 to 53	expansion through exhaust nozzle

The object of the calculations is to obtain the thrust power, the specific fuel consumption, and the heat-exchanger weight for given values of altitude, blower compression ratio, maximum temperature, heat-exchanger effectiveness, and heat-exchanger cross-sectional area. The method of calculation is to compute the changes in the condition of the fluid in each of the component parts of the cycle.

In the equations given herein, the condition of the fluid at any station is generally given in terms of its total, or stagnation, enthalpy E and its total, or stagnation, pressure H. Exceptions are the equations that contain station 0, which is in front of the airplane; station 53, which is behind the airplane; and stations 21 and 22, which are inside the heat exchanger. At these stations the velocities must be known.

The process of taking the air into the airplane is assumed to be isentropic. The total pressure and the total enthalpy of the air as it goes into the compressor are given by the relations

$$H_{10} = p_o \left(1 + \frac{\gamma - 1}{2\gamma} \frac{\rho_o V_o^2}{p_o}\right)^{\frac{\gamma}{\gamma-1}}$$

and

$$E_{10} = E_o \left(1 + \frac{\gamma - 1}{2\gamma} \frac{\rho_o V_o^2}{p_o}\right)$$

The pressure just behind the compressor is

$$H_{21} = H_{20} = R_c H_{10}$$

The enthalpy that the fluid would have after compression, if the process of compression were isentropic, is

$$E_{20} = E_{10}(R_c)^{\frac{\gamma-1}{\gamma}}$$

The efficiency of compression is by definition

$$\eta_c = \frac{E_{20} - E_{10}}{E_{21} - E_{10}}$$

(A value of 85 percent was used herein for η_c.) The enthalpy after compression is accordingly

$$E_{21} = \frac{E_{20} - (1 - \eta_c)E_{10}}{\eta_c}$$

Between stations 21 and 22 the air passes through the heat exchanger. The changes in pressure and enthalpy that are effected in the heat exchanger are discussed separately.

Between stations 22 and 30 combustion occurs. The enthalpy at station 30 is obtained by assigning a maximum permissible value to the fluid temperature. The pressure at station 30 is taken as equal to that at station 22. In other words, no account is taken of losses of pressure in the combustion system due to friction, changes in passage shape and size, and the addition of heat. In practice, the sum of these losses may be fairly large but not enough data are available to make satisfactory calculations of the losses. If the open cross-sectional area of the combustion chamber is approximately the same as the open cross-sectional area (for one fluid) of the heat exchanger, the pressure loss due to the addition of heat to a compressible fluid can be shown to be of the order of 1 percent of the pressure at station 22.

The fuel-air ratio is

$$\frac{W_f}{W} = \frac{E_{30} - E_{22}}{19{,}000 - E_{30} + E_{22}}$$

where the enthalpies refer to the mixture of air and fuel.

The condition is made herein that the working fluid issues from the exit nozzle at free-stream speed as well as at free-stream pressure. This condition necessitates the use of trial-and-error methods for the calculation of conditions between stations 30 and 53. The procedure is to use an estimated value of E_{43} in calculating conditions at the other stations that follow station 30. The enthalpy at station 53 is given by the relation for the enthalpy change that is required to accelerate the fluid from zero speed to free-stream speed

$$E_{43} - E_{53} = \frac{V_o^2}{2gJ}$$

where an estimated value of E_{43} is used and where E_{53} is static, not total, enthalpy. The required pressure at station 43 is

$$H_{43} = p_0 \left(\frac{E_{43}}{E_{53}}\right)^{\frac{\gamma}{\gamma-1}}$$

The pressure at stations 41 and 42 is that at station 43 with the absolute value of the pressure loss in the heat exchanger added. The enthalpy at station 42 is

$$E_{42} = E_{30}\left(\frac{H_{42}}{H_{30}}\right)^{\frac{\gamma-1}{\gamma}}$$

The enthalpy at station 41 is given by the equation defining the turbine efficiency

$$\eta_T = \frac{E_{30} - E_{41}}{E_{30} - E_{42}}$$

(In the present calculations, η_T was given a value of 85 percent.) The enthaply at station 43 is therefore

$$E_{43} = E_{41} - E_{22} + E_{21}$$

and this value of E_{43} must check with the estimated value previously used.

The total work done by the turbine is $E_{30} - E_{41}$. Of this quantity, $E_{21} - E_{10}$ is the work done on the compressor and the remainder is the work done on the propeller. The work done on the propeller is then multiplied by the propeller efficiency, which in the present paper was 85 percent.

Heat-Exchange System

An analysis is given of the heat-exchange system that includes the equations for calculating the rate of heat transfer and the pressure drop in the heat exchangers and the method of calculating the core dimensions and the installed weights of the heat exchangers. The basic equations of heat transfer and pressure drop used in the discussion of the heat-exchange system were taken from reference 5.

The available temperature difference for heat transfer is $T_{41} - T_{21}$. The drop in the temperature of the hot fluid divided by the available difference is

$$\xi = \frac{T_{41} - T_{43}}{T_{41} - T_{21}}$$

The rise in the temperature of the cold fluid divided by the available difference is

$$\eta = \frac{T_{22} - T_{21}}{T_{41} - T_{21}}$$

As a convenience in making the calculations, the approximate relations

$$\xi = \frac{E_{41} - E_{43}}{E_{41} - E_{21}}$$

and

$$\eta = \frac{E_{22} - E_{21}}{E_{41} - E_{21}}$$

in which temperatures are replaced by enthalpies, can be used. The resulting inaccuracy is about 1 percent.

The fuel-air ratios are so small that the weight of the fuel can be neglected. The energy-balance equations for the heat transfer are therefore

$$\begin{aligned} Q &= W(E_{22} - E_{21}) \\ &= W\eta(E_{41} - E_{21}) \end{aligned}$$

$$Q = W(E_{41} - E_{43})$$

$$= W\xi(E_{41} - E_{21})$$

and the quantities ξ and η are equal.

The heat exchangers considered herein are chosen to be of the counterflow type. The mean temperature difference between the hot fluid and the cold fluid expressed as a fraction of the available temperature difference is accordingly

$$\zeta = \frac{\xi - \eta}{\log_e \dfrac{1 - \eta}{1 - \xi}}$$

For $\xi = \eta$, this expression is indeterminate. Application of L'Hospital's rule shows, however, that when $\xi = \eta$

$$\zeta = 1 - \xi$$

The rate of heat transfer is

$$Q = US\zeta \frac{\Delta E_1}{c_p}$$

where

$$\frac{1}{US} = \frac{1}{h_1 S_1} + \frac{1}{h_2 S_2}$$

and

$$h = 0.2 \left(\frac{\mu}{D}\right) R^{0.8}$$

Inasmuch as h_1 is very nearly equal to h_2 and S_1 is very nearly equal to S_2, the rate of heat transfer can be written as

$$Q = 0.1\pi\mu R^{0.8}\ LN\ \frac{\Delta E_1}{c_p}(1 - \xi)$$

If this equation for the rate of heat transfer is divided by $W\ \Delta E_1$ and 0.24 is substituted for c_p,

$$\frac{Q}{W \; \Delta E_i} = 0.052R^{-0.2} \frac{L}{D}(1 - \xi)$$

But the rate of heat transfer is also given by

$$Q = W\xi \; \Delta E_i$$

so that

$$\frac{Q}{W \; \Delta E_i} = \xi = 0.052R^{-0.2} \frac{L}{D}(1 - \xi)$$

and

$$\xi = \frac{0.052R^{-0.2} \frac{L}{D}}{1 + 0.052R^{-0.2} \frac{L}{D}}$$

The pressure drop due to friction in the heat exchanger is given by the relation

$$\frac{\Delta p}{q} = 0.184R^{-0.2} \frac{L}{D}$$

where the value of q is that inside the heat-exchanger tubes. The relation between $\Delta p/q$ and ξ is therefore

$$\frac{\Delta p}{q} = 3.55\frac{\xi}{1 - \xi}$$

The value of $\Delta p/q$ therefore depends only on the value of ξ, and the value of Δp can be obtained when ξ and q are known. The value of q is given by the relation

$$q = \frac{1}{2}\rho V^2 = \frac{R}{2g}\left(\frac{W}{A}\right)^2 \frac{T + 460}{p}$$

where R is the gas constant,

$$R = \frac{p}{g\rho(T + 460)}$$

One-fifth of the value of q was added to the pressure drop calculated as just indicated in order to take into account the loss occurring at the exit of the heat exchanger.

The calculations in the present paper are made on the basis of stagnation pressure and stagnation enthalpy. The equations just given for calculating Δp are strictly valid only for calculating the static-pressure drop due to friction. Inasmuch as the Mach number of the flow in the heat exchangers is very small, however, the drop in total pressure due to friction is very nearly equal to the drop in static pressure due to friction. Because of the rather small temperature differences in the heat exchangers between the tube walls and the fluids, the changes in total pressure due to the transfer of heat to and from a compressible fluid are in most cases quite small. Furthermore, the effect of the loss in total pressure due to the acceleration of the cold fluid that results from heat transfer is partly offset by the effect of the gain in total pressure due to the deceleration of the hot fluid. The change in total pressure in the heat exchanger was therefore considered herein to be equal to the change in static pressure due to friction.

For a given value of A/W, the Reynolds number is found from the relation

$$R = \frac{D}{g\mu} \frac{W}{A}$$

The length of the heat-exchanger tubes is given by

$$\frac{L}{D} = 19.3 \left(\frac{D}{g\mu}\right)^{0.2} \left(\frac{W}{A}\right)^{0.2} \frac{\xi}{1 - \xi}$$

The heat exchangers considered herein are made of circular tubes. The tubes are made of stainless steel and are of 0.25-inch inside diameter and 0.020-inch wall thickness. The center-to-center spacing of the tubes is

0.34 inch and there are 1250 tubes per square foot of heat-exchanger frontal area. One fluid flows through the tubes and the other flows over the outside of the tubes in the direction of the tube axis. Both fluids flow through passages of equal hydraulic diameter and of equal cross-sectional area. For each fluid, the ratio of open frontal area to the total frontal area of the heat exchanger is 0.426.

The specific weight of the core is 71 pounds per cubic foot of heat exchanger. The weight of the core is given in pounds per pound of air per second by the expression $3.47 \frac{L}{D} \frac{A}{W}$ or $67 \left(\frac{D}{g\mu}\right)^{0.2} \left(\frac{A}{W}\right)^{0.8} \left(\frac{\xi}{1 - \xi}\right)$. The weight of the heat-exchanger installation (core, shell, ducting) is taken herein as 1.5 times the calculated weight of the heat-exchanger core.

In order to show graphically the interrelations of the installed weight of the heat exchanger and the effectiveness, the cross-sectional area, and the length of the heat exchanger, figure 39 is presented. Although the quantity $D/g\mu$ is different for the hot and the cold sides of the heat exchanger, it enters in the equations only to the 0.2 power. In figure 39, a constant value of $D/g\mu$ of 1100 square feet-seconds per pound was used. The tube length for any of the heat exchangers discussed herein may be determined from figure 39.

Effect of Altitude

The effect of altitude, or atmospheric density, on the cross-sectional area and the weight of the heat exchanger can be obtained as follows: In the analysis given earlier, the heat-exchanger effectiveness ξ is shown to be related to the pressure drop and the dynamic pressure in the heat exchanger by

$$\frac{\xi}{1 - \xi} \propto \frac{\Delta p}{q}$$

For constant values of ξ and Δp, therefore, q must be constant but

$$q = \frac{1}{2}\rho V^2$$

$$= \frac{1}{2g^2}\left(\frac{W}{A}\right)^2 \frac{1}{\rho}$$

For constant q, therefore, the relation

$$\frac{A}{W} \propto \left(\frac{1}{\rho}\right)^{0.5}$$

must be satisfied. (Although ρ is in this expression the density in the heat exchanger, it is proportional to atmospheric density for constant compressor efficiency and compression ratio.) The weight of the heat exchanger per pound of air per second has been shown to be given, for constant values of $D/g\mu$, by the relation

$$\frac{\text{Weight}}{W} \propto \left(\frac{A}{W}\right)^{0.8} \frac{\xi}{1 - \xi}$$

For constant ξ therefore,

$$\frac{\text{Weight}}{W} \propto \left(\frac{1}{\rho}\right)^{0.4}$$

APPENDIX C

SOURCE OF THERMODYNAMIC DATA FOR AIR

The data on the thermodynamic properties of air given in reference 6 were used in making the computations for the present paper. A table of the enthalpy changes associated with isentropic pressure changes is presented in reference 6. The use of this table eliminates much tedious calculation or interpolation on a Mollier chart. The range of initial temperature for which the table is given is from 900° F to 2000° F. When the initial temperature in any calculation was 900° F or more, the table was used. When the initial temperature was less than 900° F, values of γ were used that were obtained from the plots given in reference 6 of c_p and c_v against temperature. When it was necessary to find the enthalpy corresponding to a given value of the temperature, the following relation from reference 6 was used:

$$E = \int_0^{T + 460} c_p \, dT$$

$$= 0.239202(T + 460) + 1.634 \times 10^{-6}(T + 460)^2$$

$$+ 1.104 \times 10^{-9}(T + 460)^3 \text{ Btu per pound}$$

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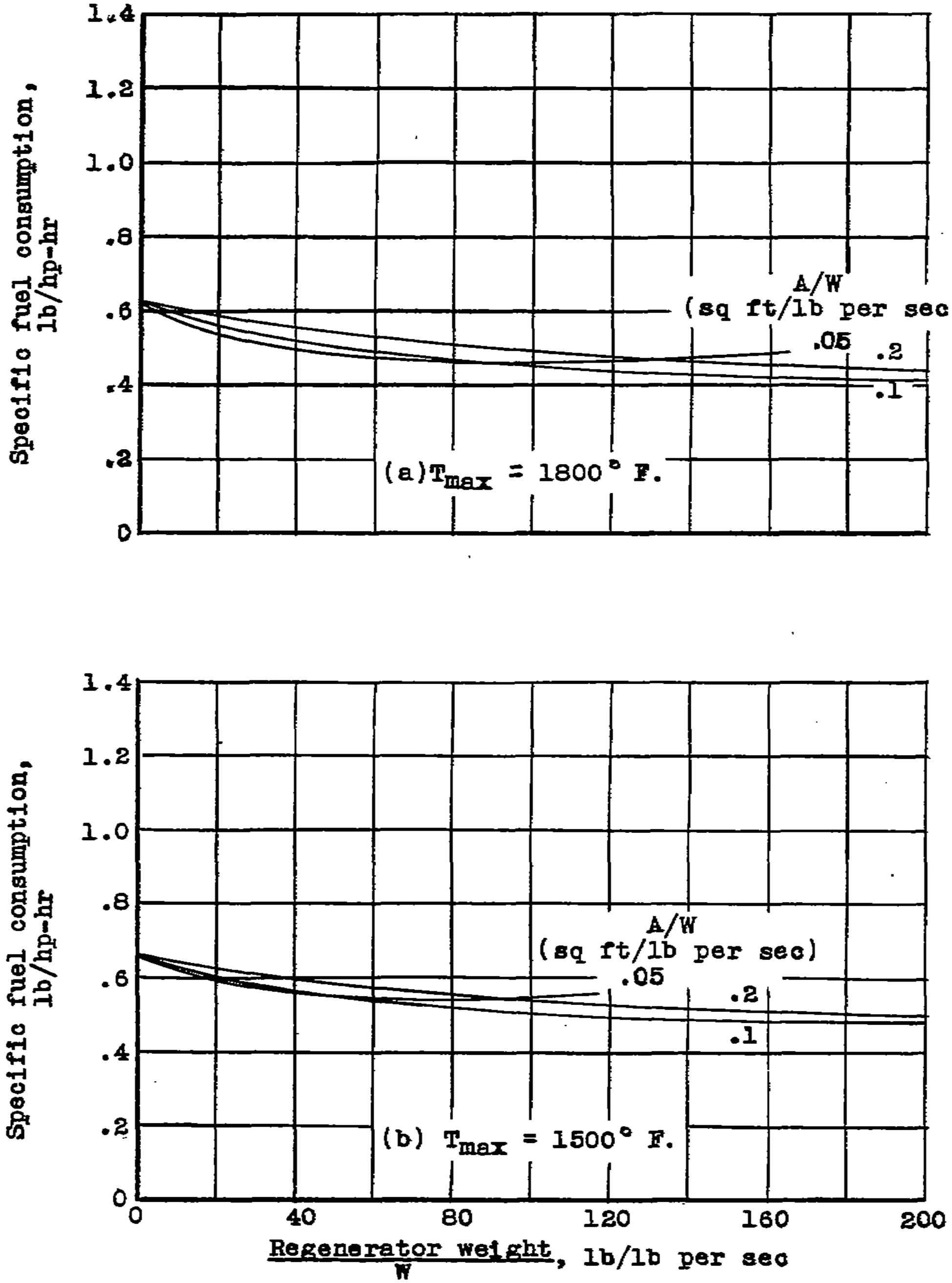

Figure 4.- Variation of specific fuel consumption with heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 6.

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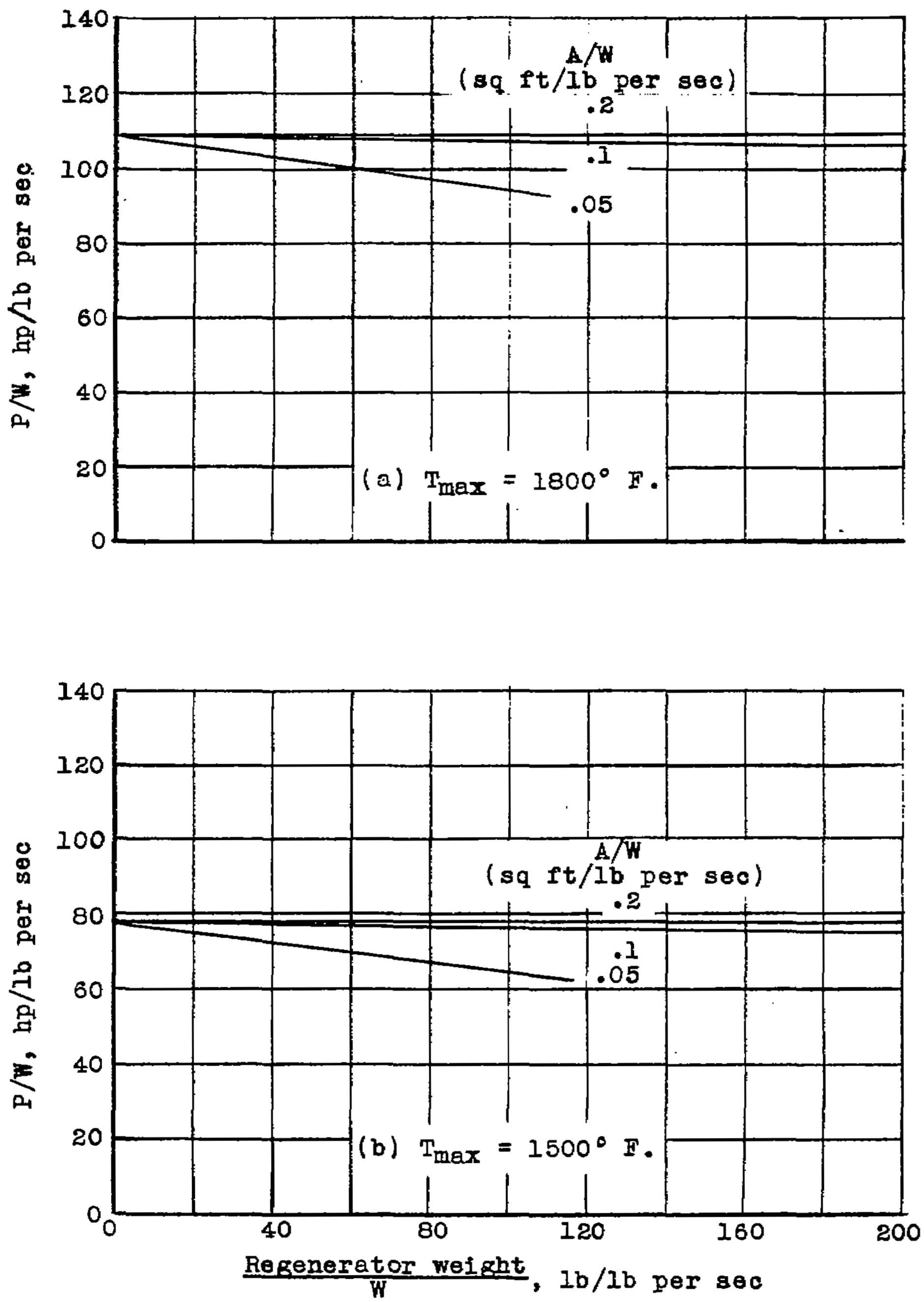

Figure 5.- Variation of thrust power per pound of air per second with heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 6.

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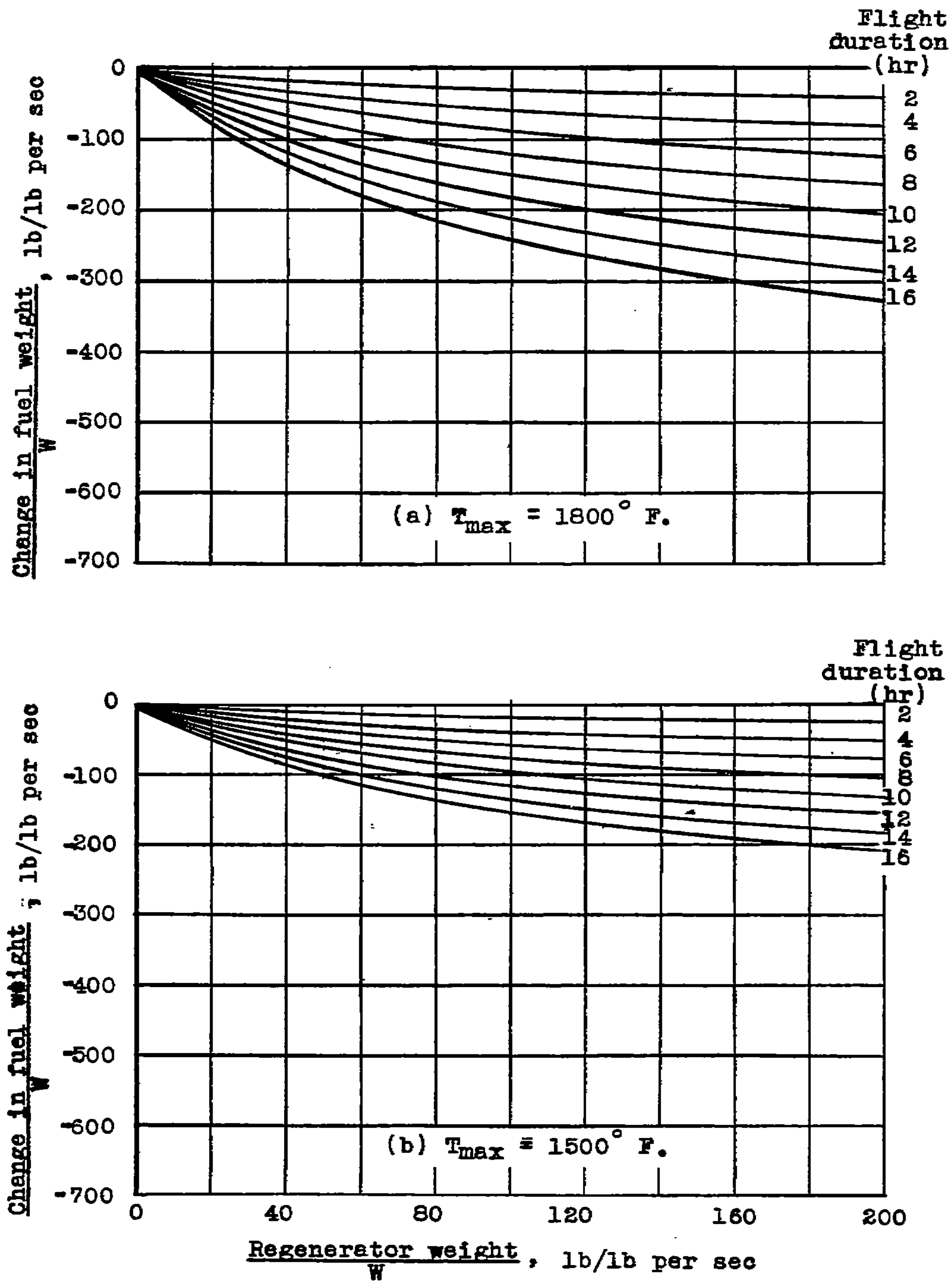

Figure 6.- Change in fuel weight per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 6; A/W = 0.2 square foot per pound of air per second.

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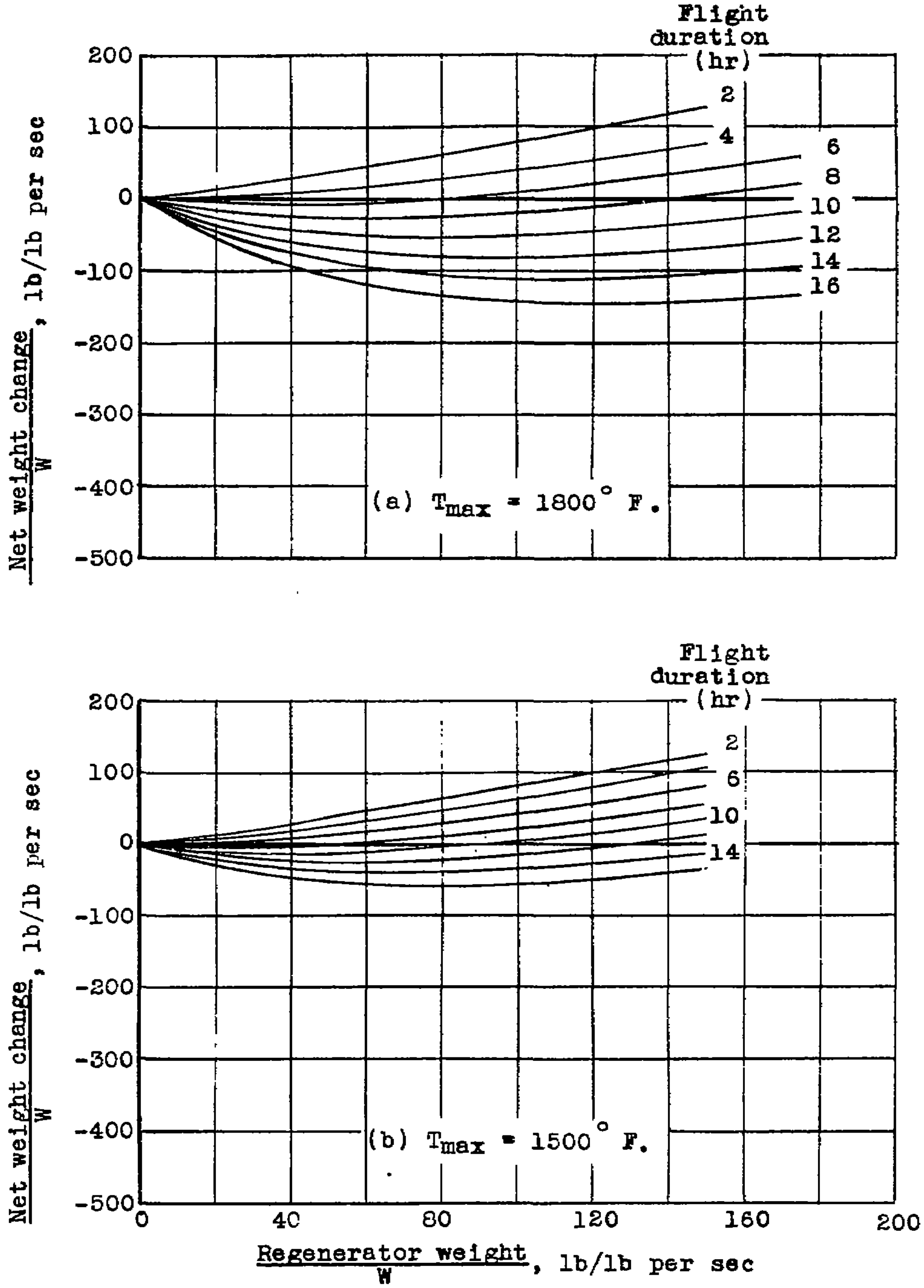

Figure 7.- Net change in weight of power plant and fuel per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 6; A/W = 0.2 square foot per pound of air per second.

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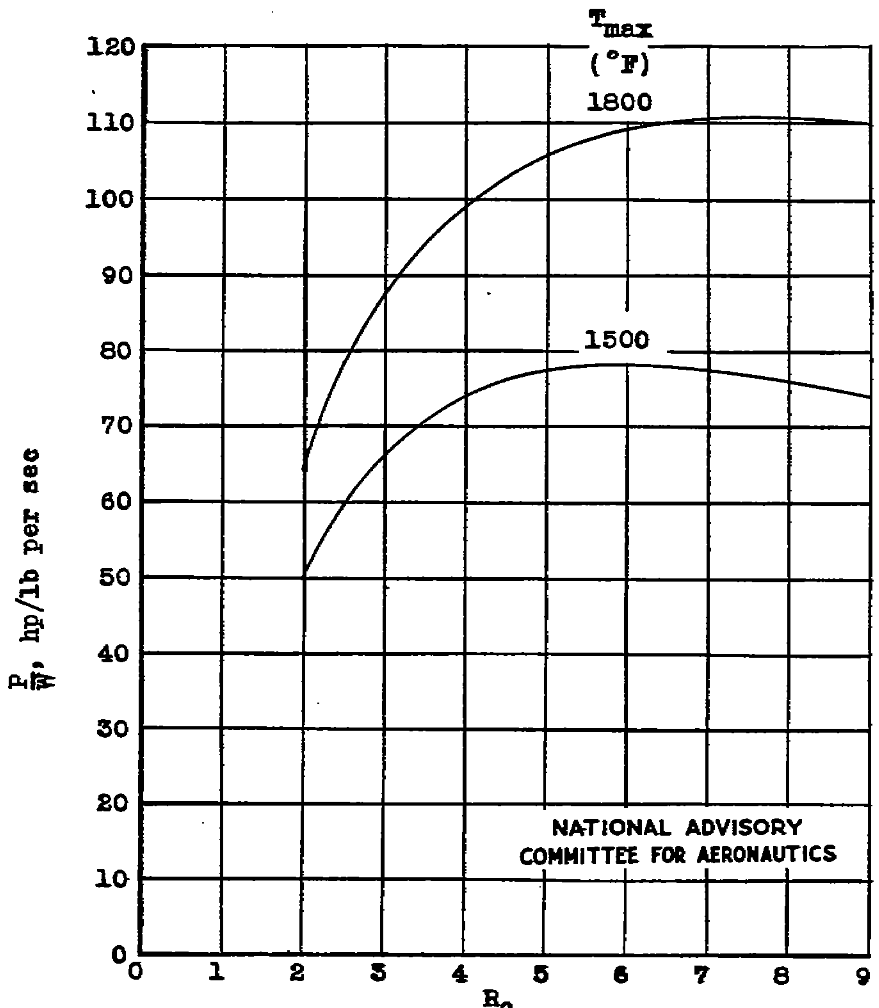


Figure 8.- Variation of thrust power per pound of air per second with compression ratio. Non-regenerative system. Altitude, sea level.

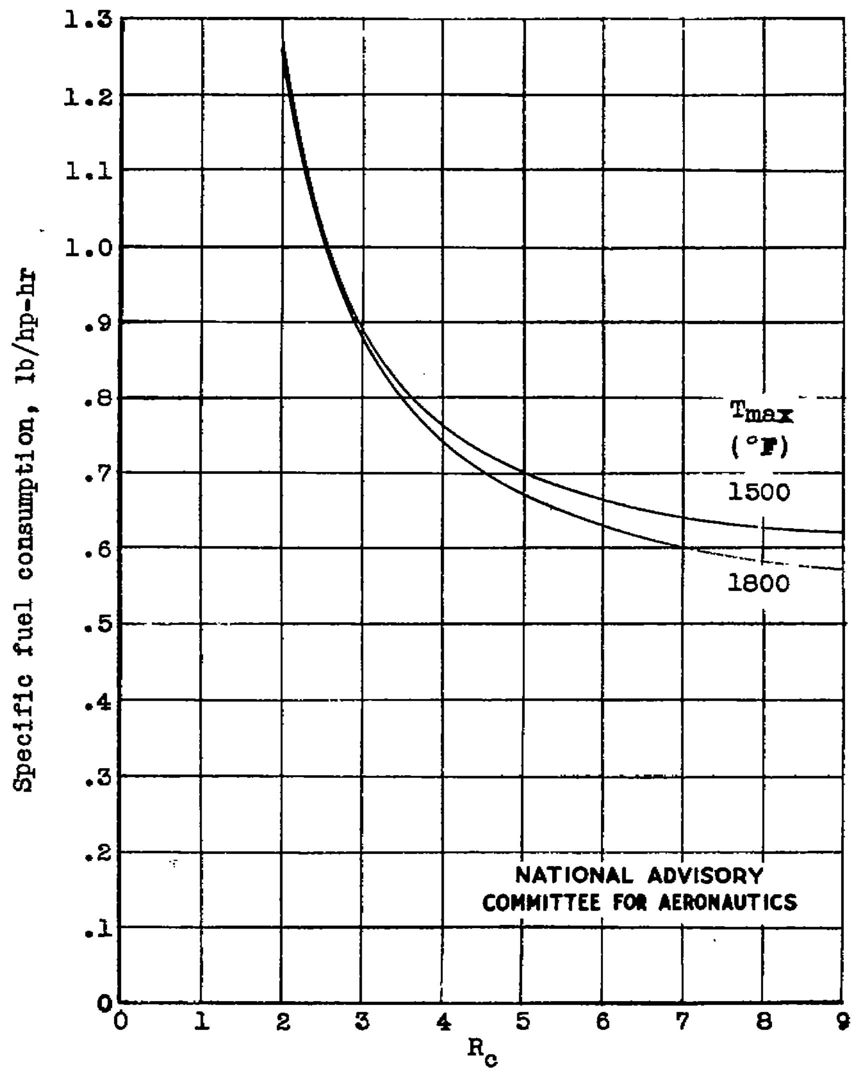
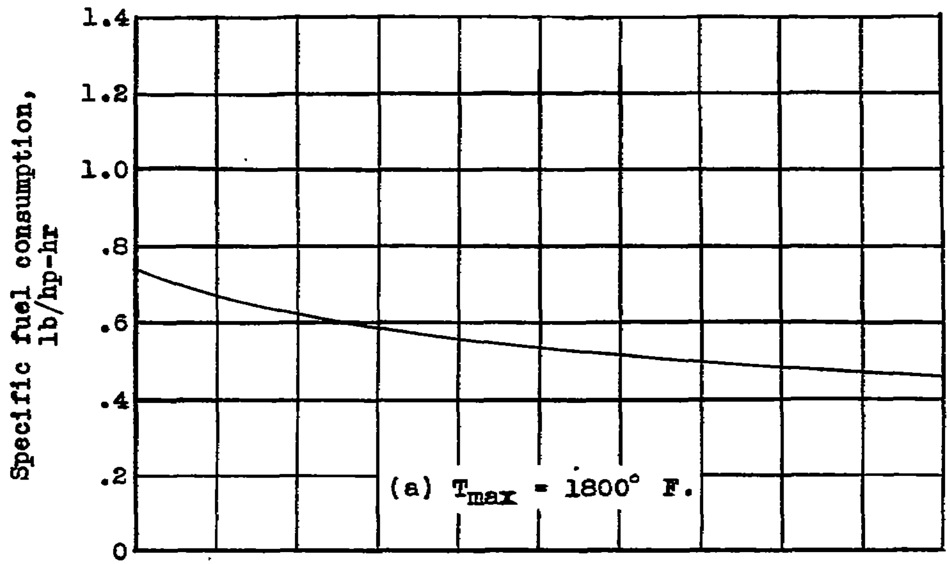


Figure 9.- Variation of specific fuel consumption with compression ratio. Nonregenerative system. Altitude, sea level.



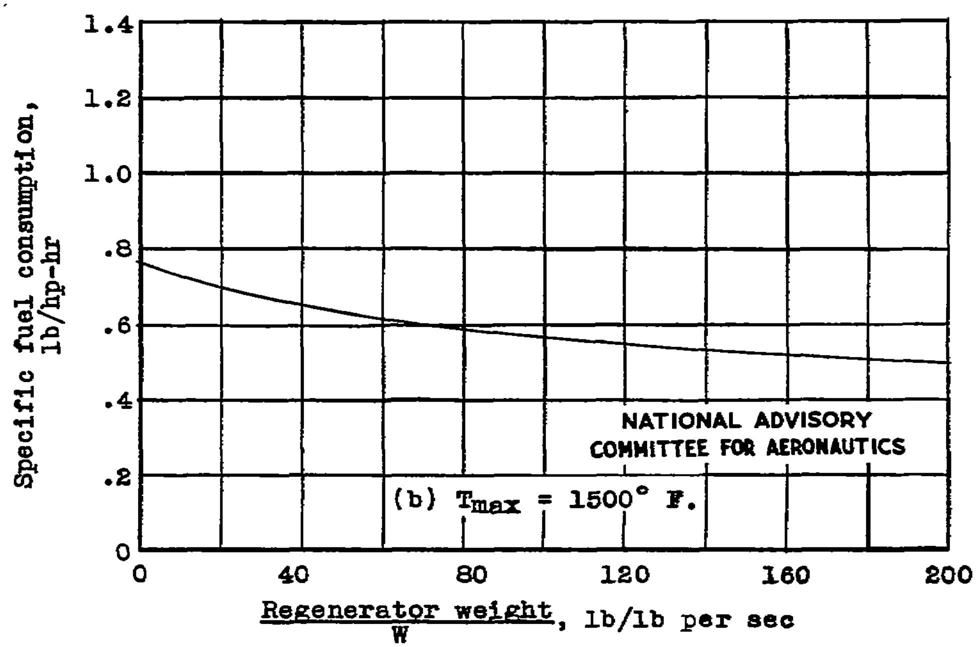


Figure 10.- Variation of specific fuel consumption with heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 4; A/W = 0.2 square foot per pound of air per second.

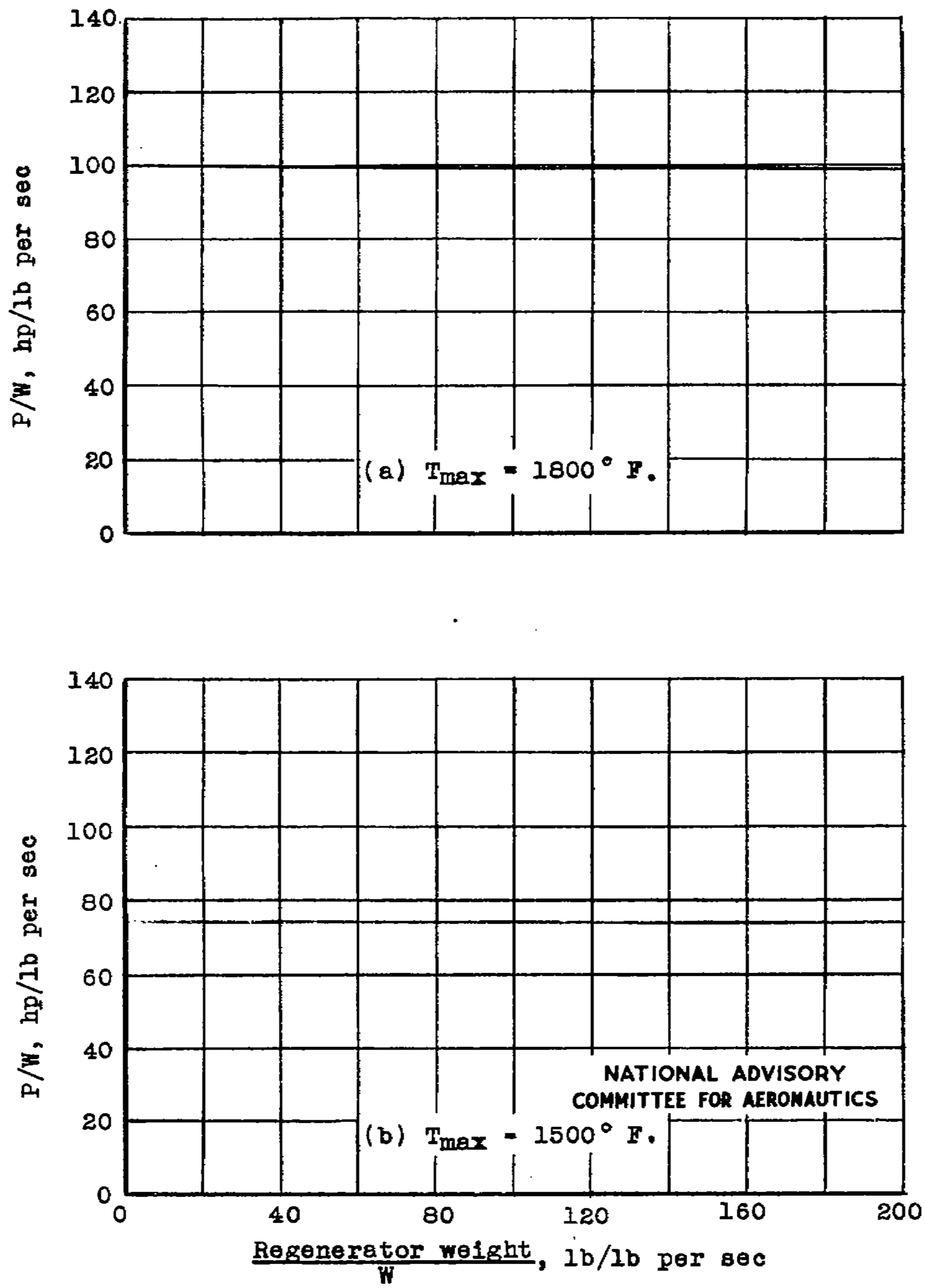


Figure 11.- Variation of thrust power per pound of air per second with heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 4; A/W = 0.2 square foot per pound of air per second.

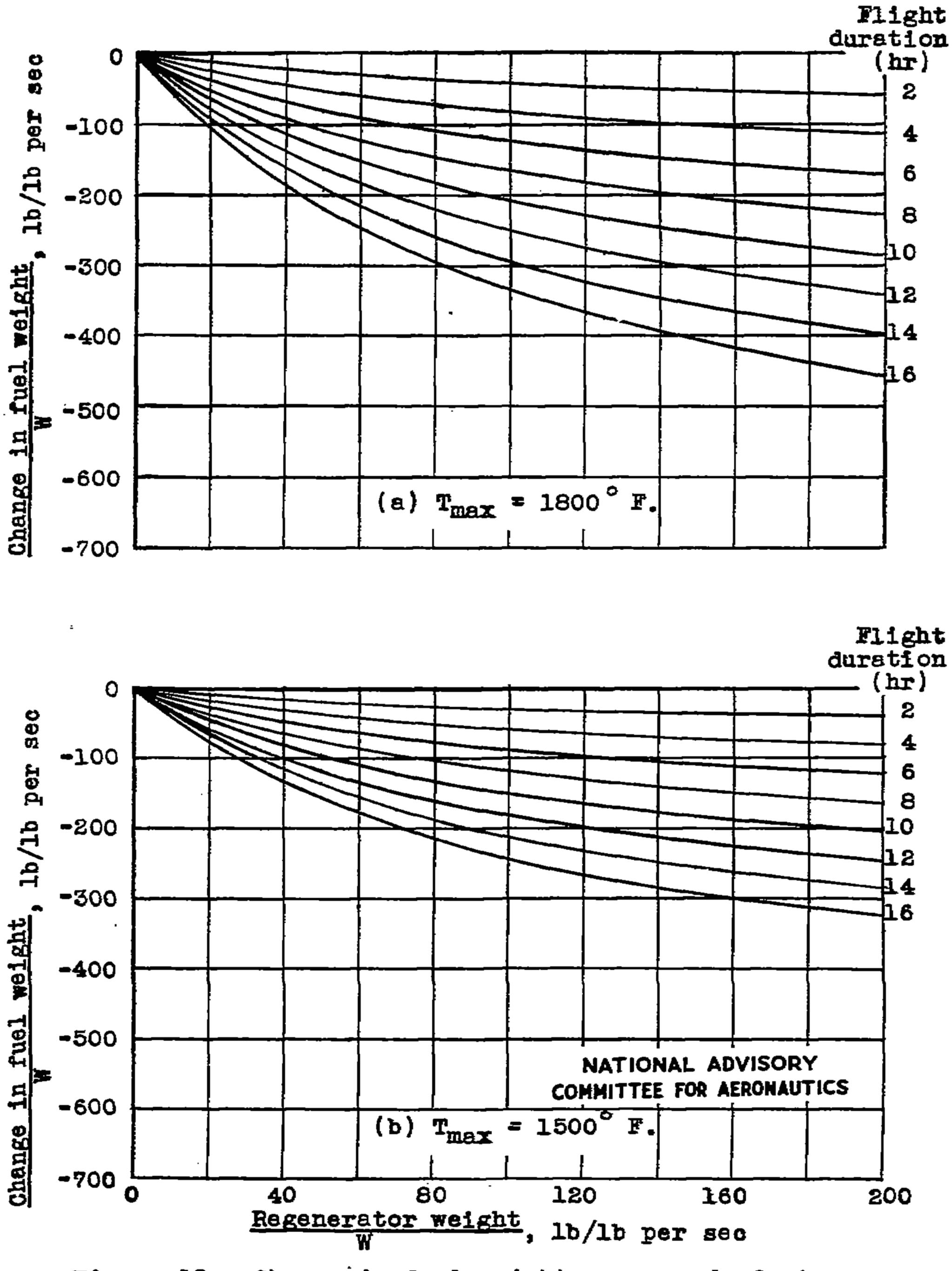


Figure 12.- Change in fuel weight per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 4; A/W = 0.2 square foot per pound of air per second.

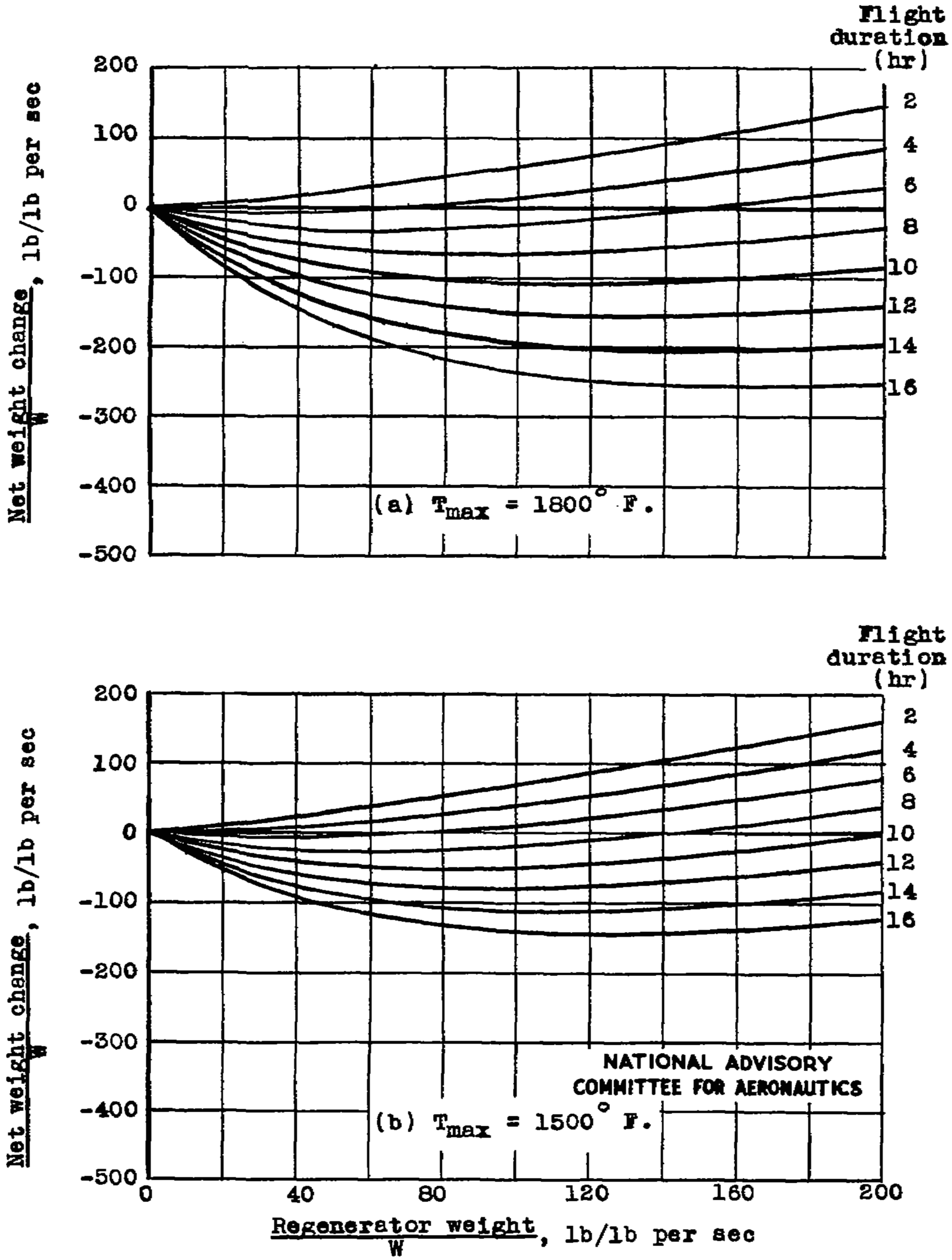


Figure 13.- Net change in weight of power plant and fuel per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 4; A/W = 0.2 square foot per pound of air per second.

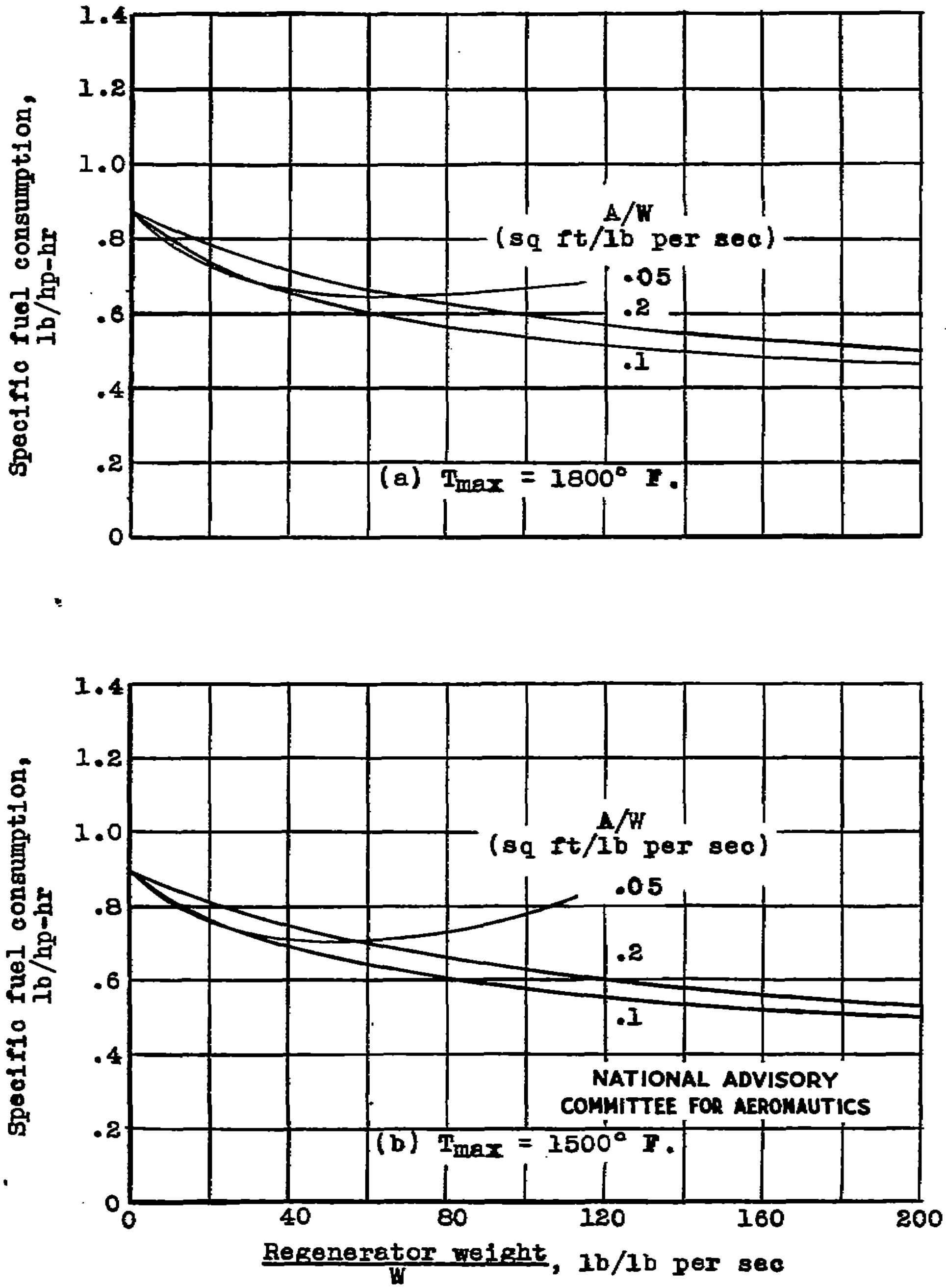


Figure 14.- Variation of specific fuel consumption with heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 3.

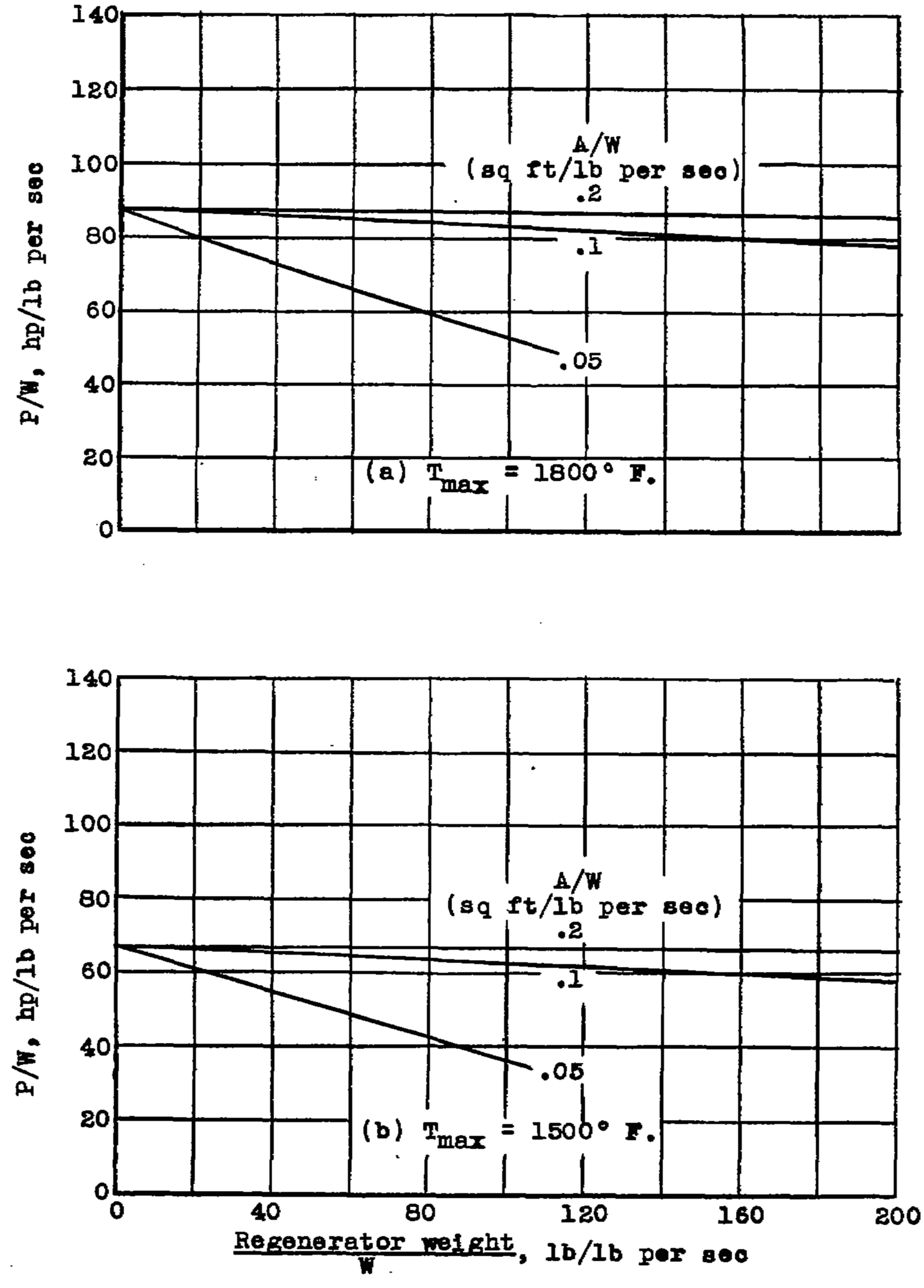


Figure 15.- Variation of thrust power per pound of air per second with heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 3.

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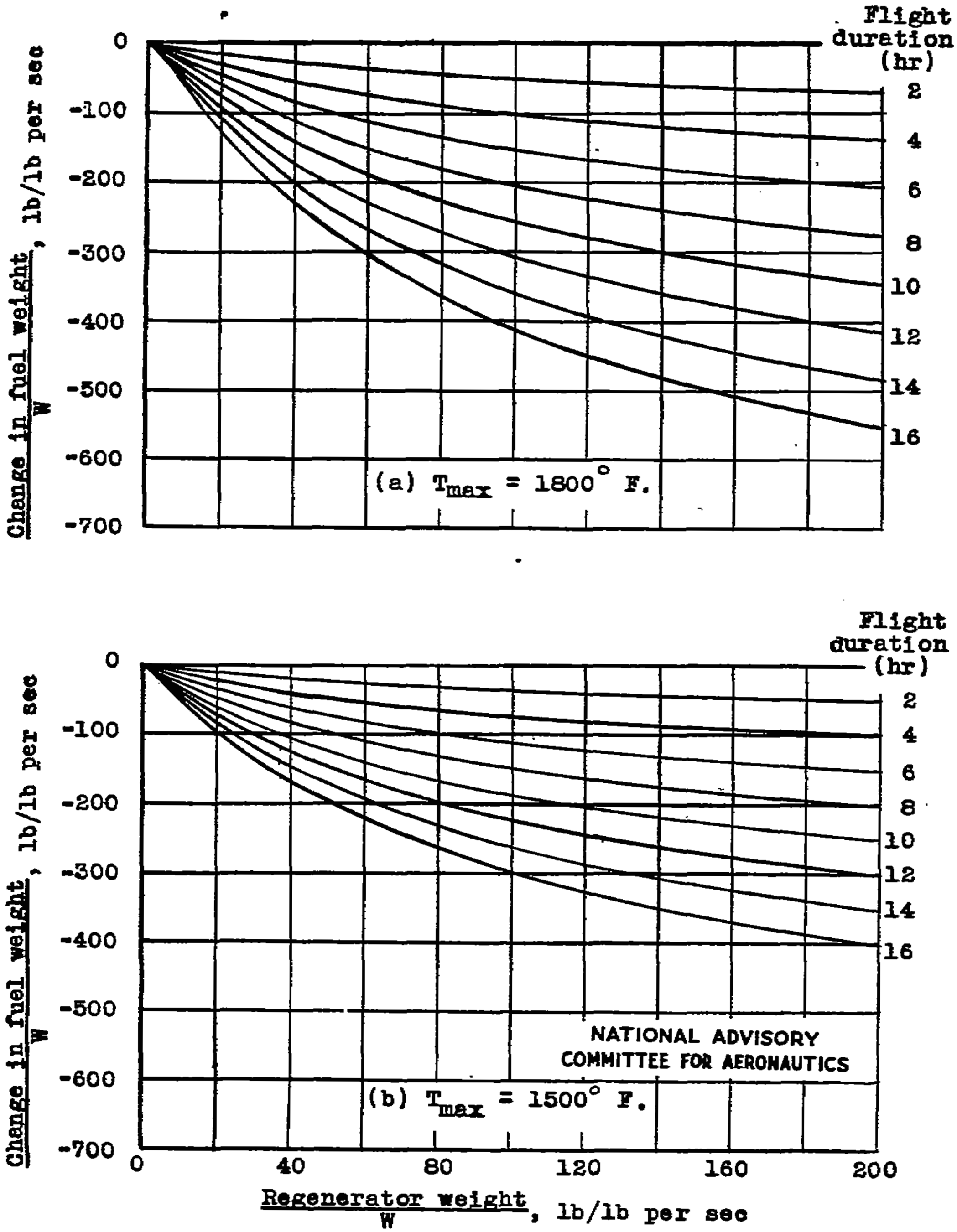


Figure 16.- Change in fuel weight per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 3; A/W = 0.2 square foot per pound of air per second.

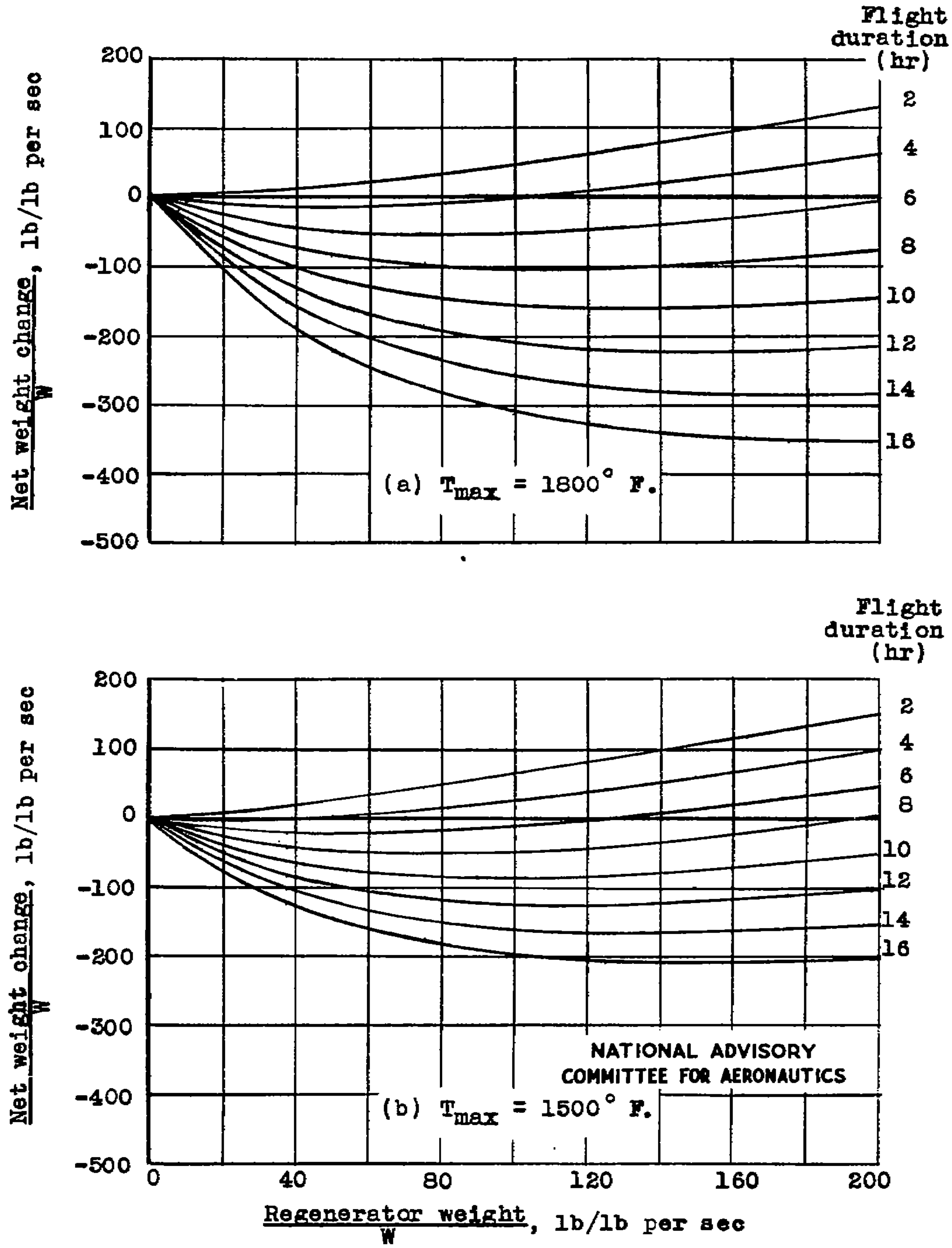


Figure 17.- Net change in weight of power plant and fuel per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, sea level; $R_c = 3$; $A/W = 0.2$ square foot per pound of air per second.

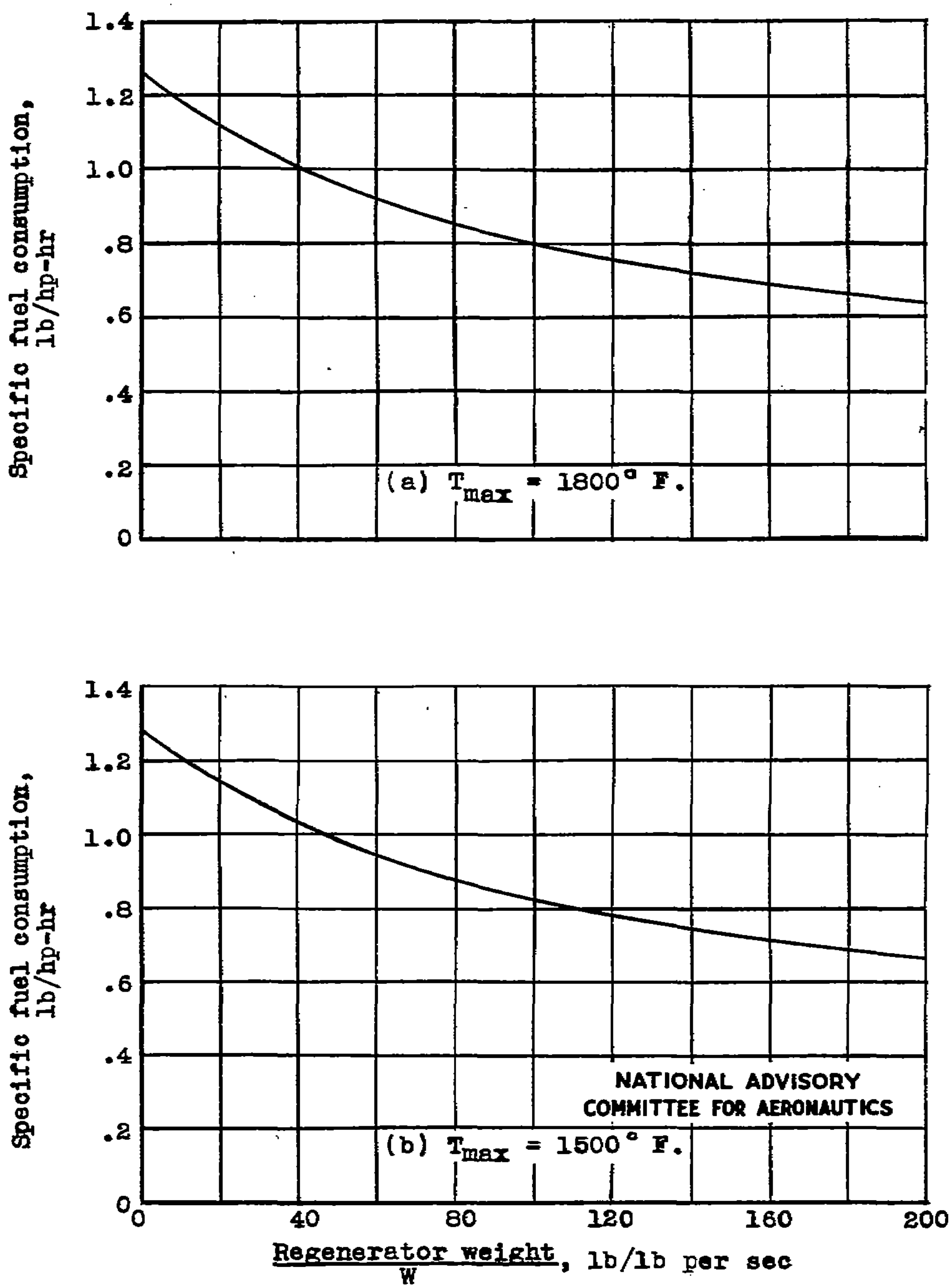


Figure 18.- Variation of specific fuel consumption with heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 2; A/W = 0.2 square foot per pound of air per second.

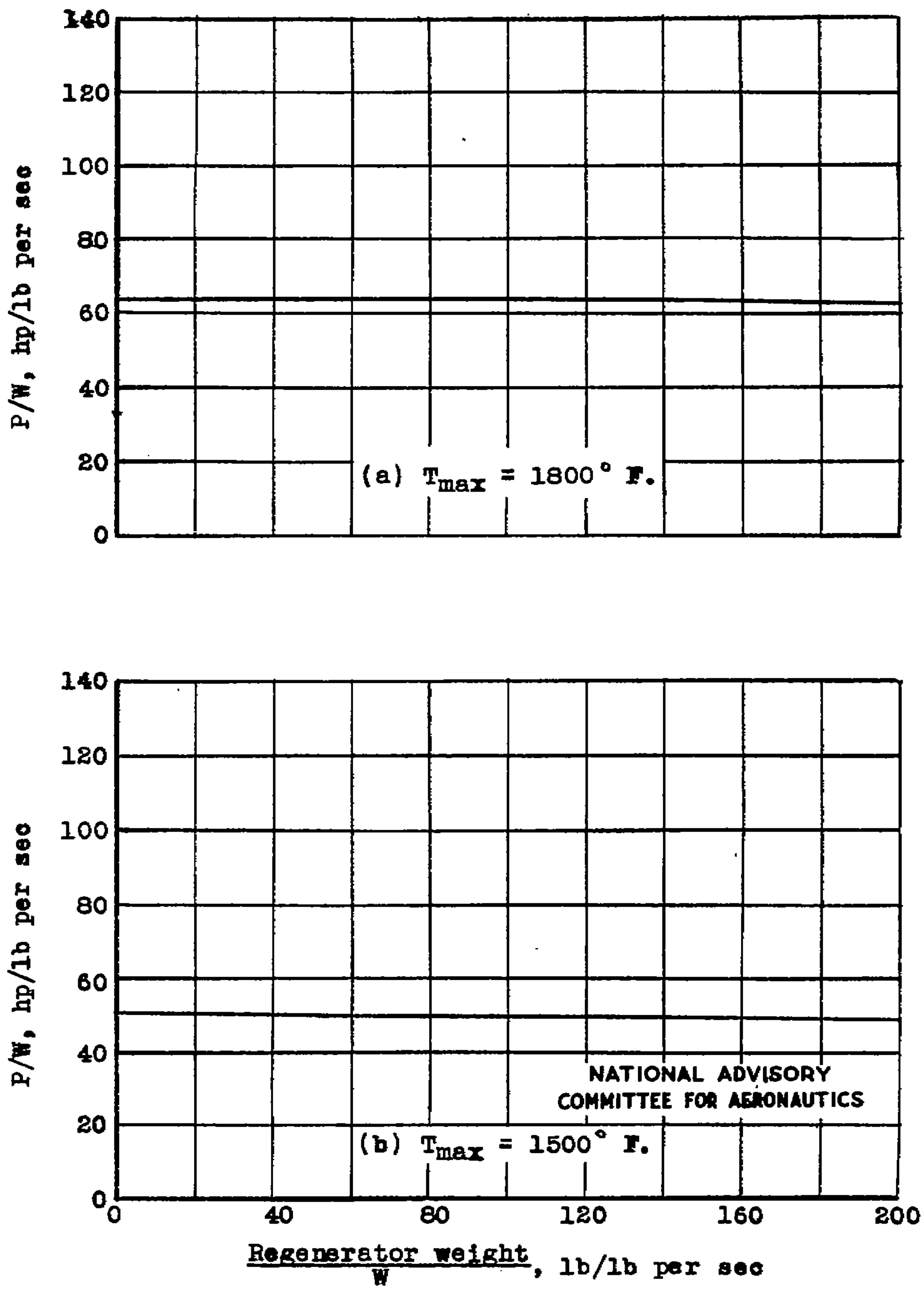
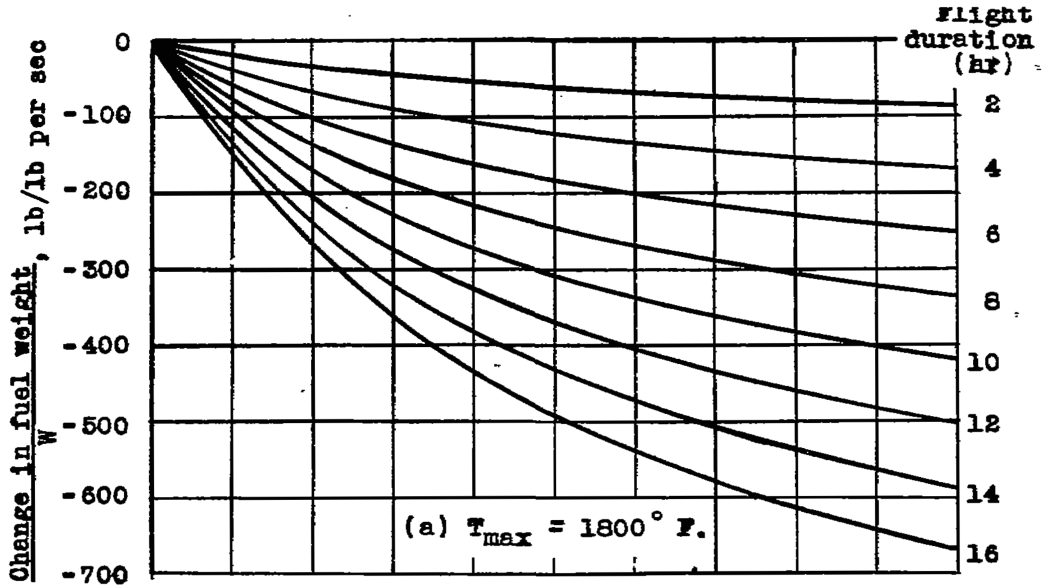


Figure 19.- Variation of thrust power per pound of air per second with heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 2; A/W = 0.2 square foot per pound of air per second.



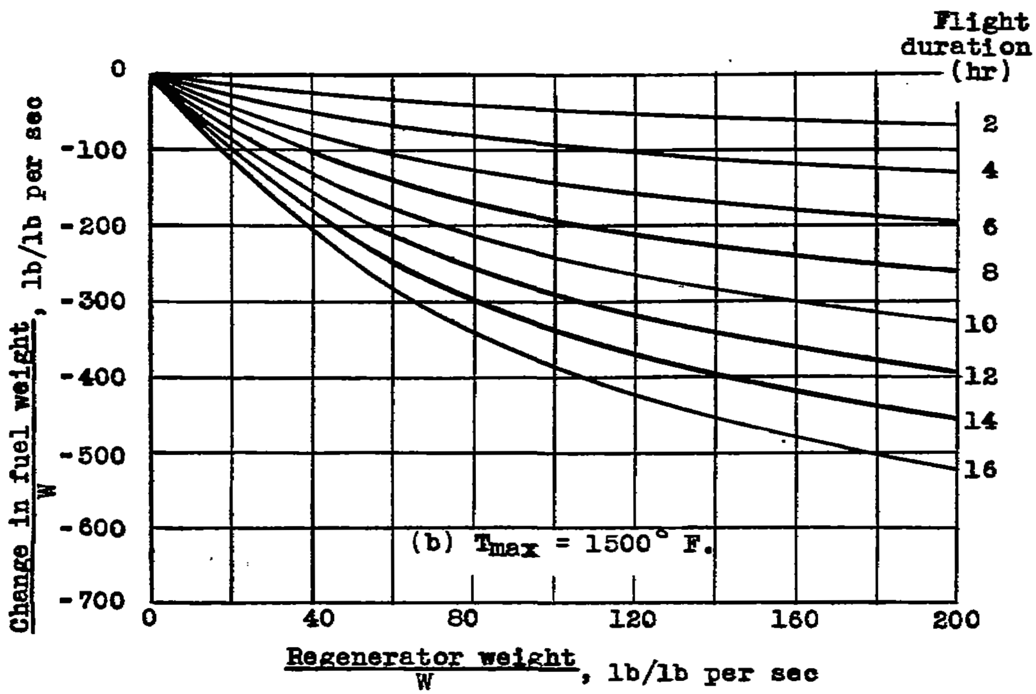


Figure 20.- Change in fuel weight per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, sea level; $R_c = 2$; A/W = 0.2 square foot per pound of air per second.

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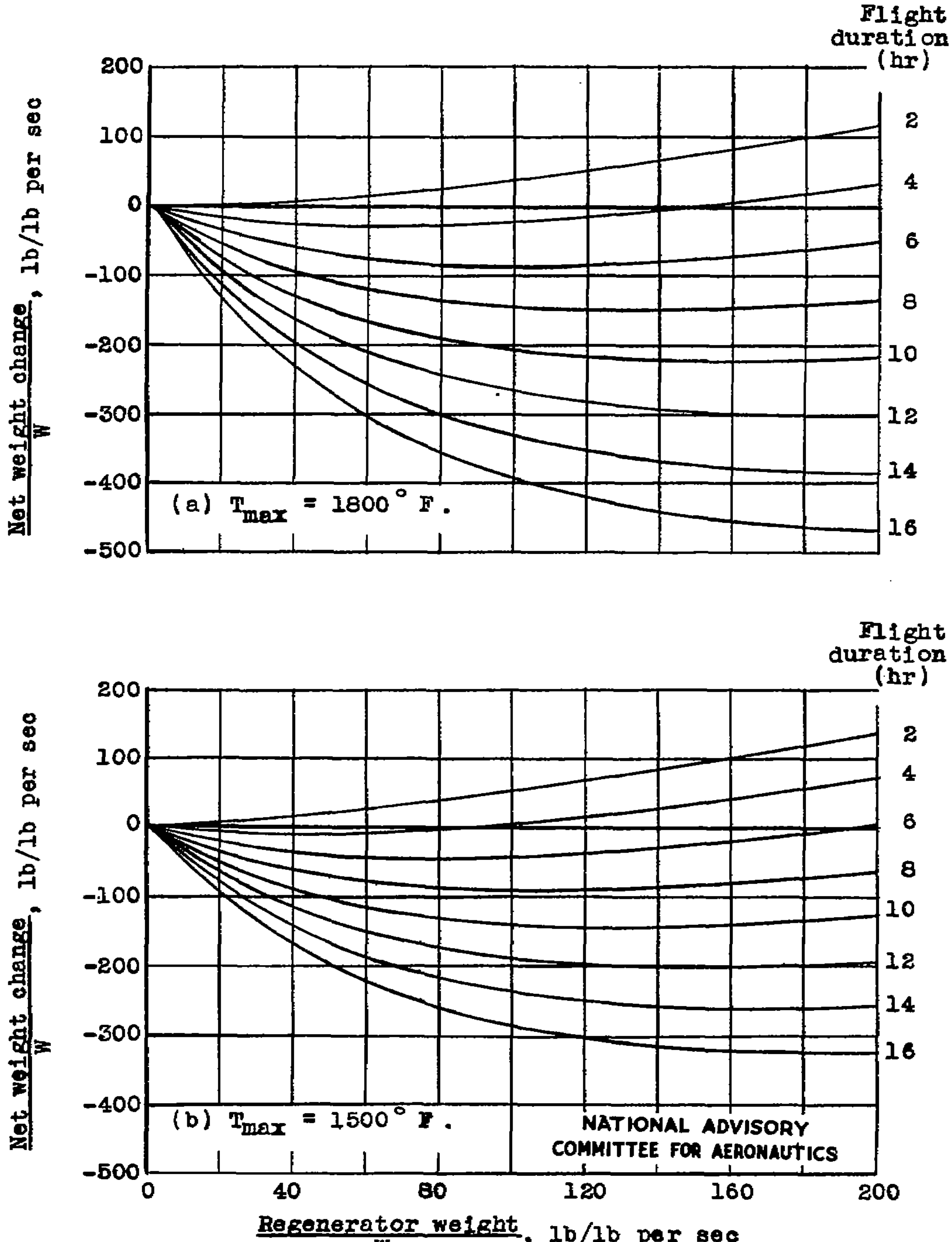


Figure 21.- Net change in weight of power plant and fuel per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, sea level; R_c = 2; A/W = 0.2 square foot per pound of air per second.

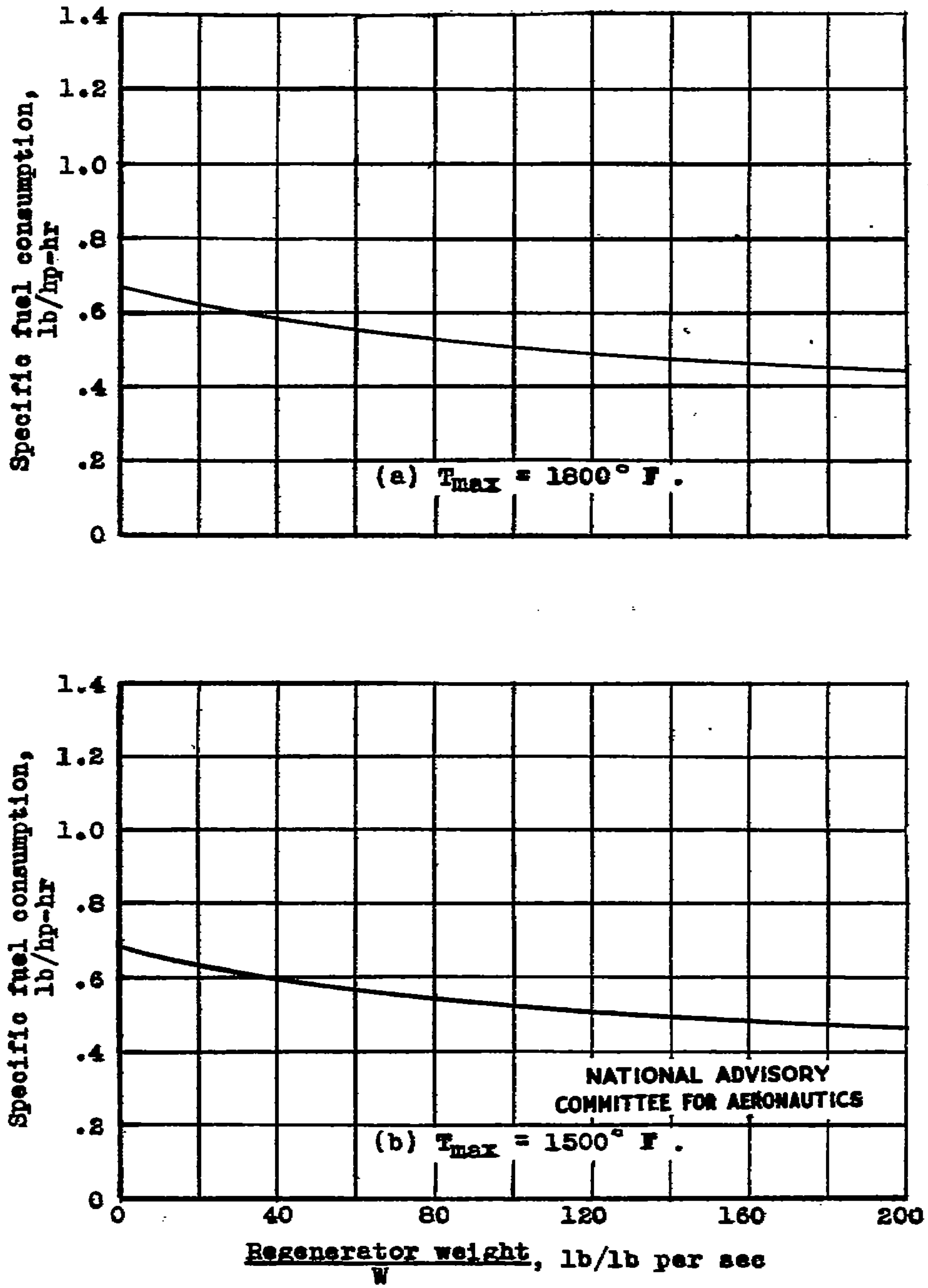


Figure 22.- Variation of specific fuel consumption with heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; $R_c = 4$; $A/W = 0.3$ square foot per pound of air per second.

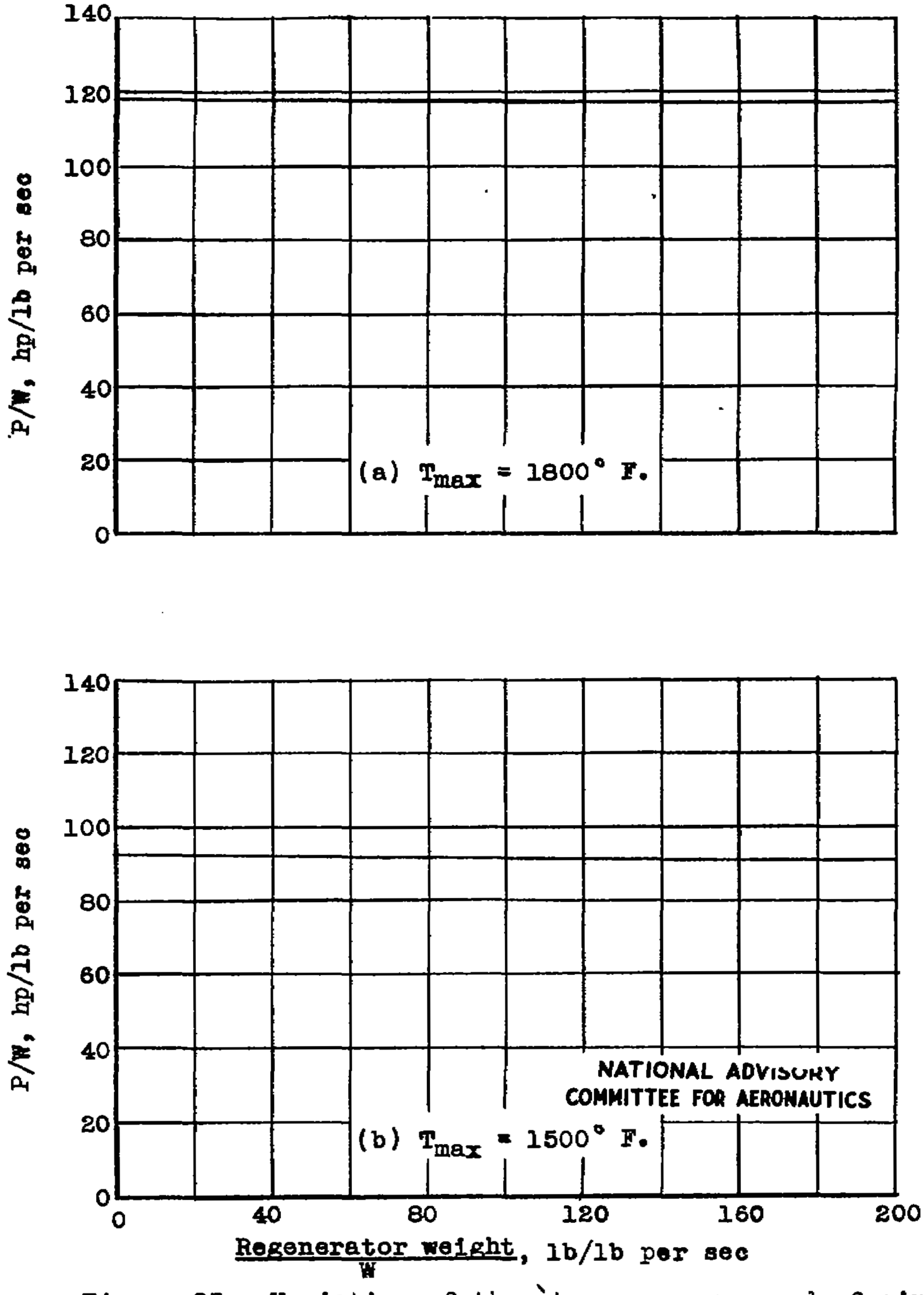


Figure 23.- Variation of thrust power per pound of air per second with heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; R_c = 4; A/W = 0.3 square foot per pound of air per second.

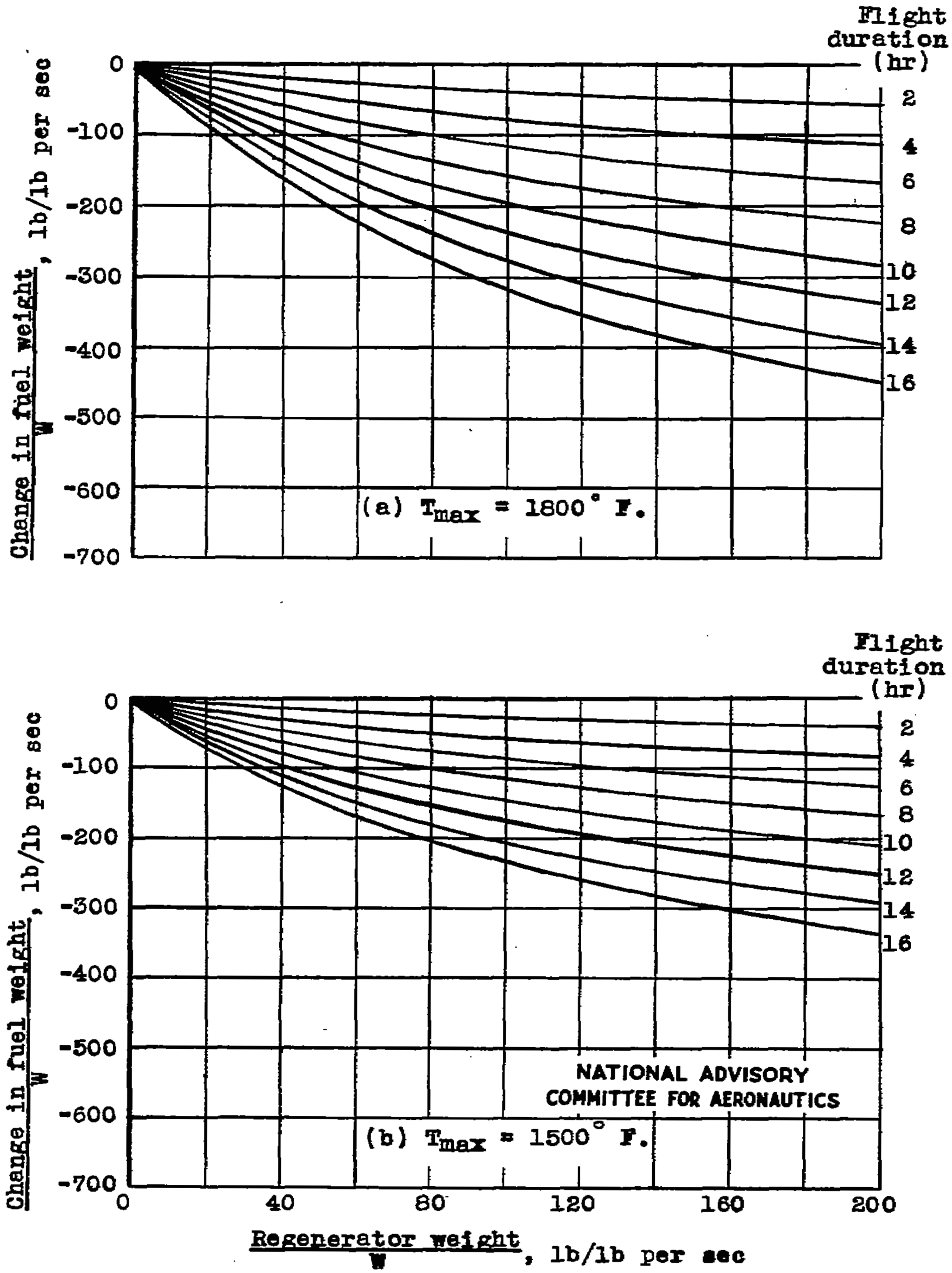


Figure 24.- Change in fuel weight per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; R_c = 4; A/W = 0.3 square foot per pound of air per second.

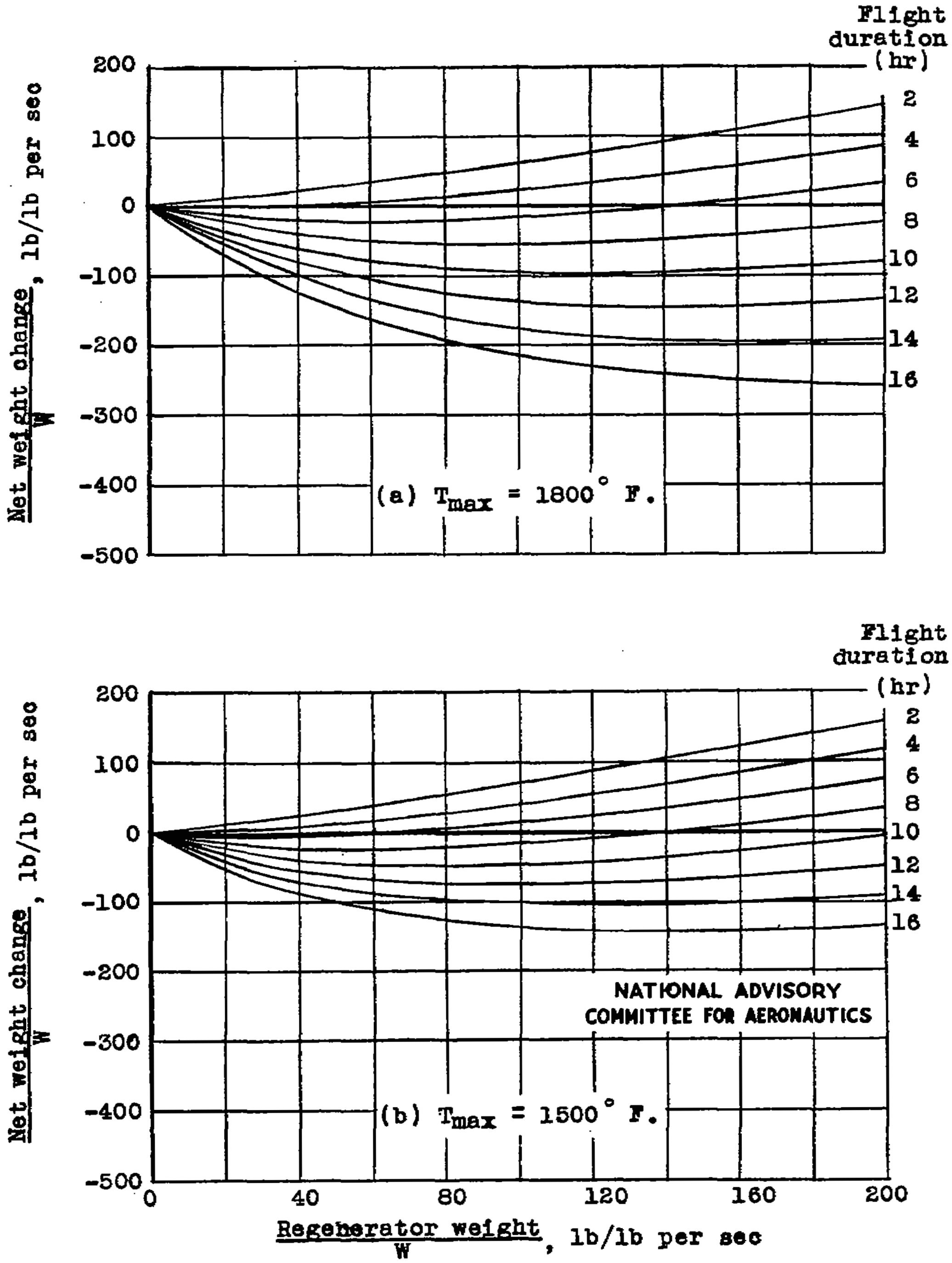
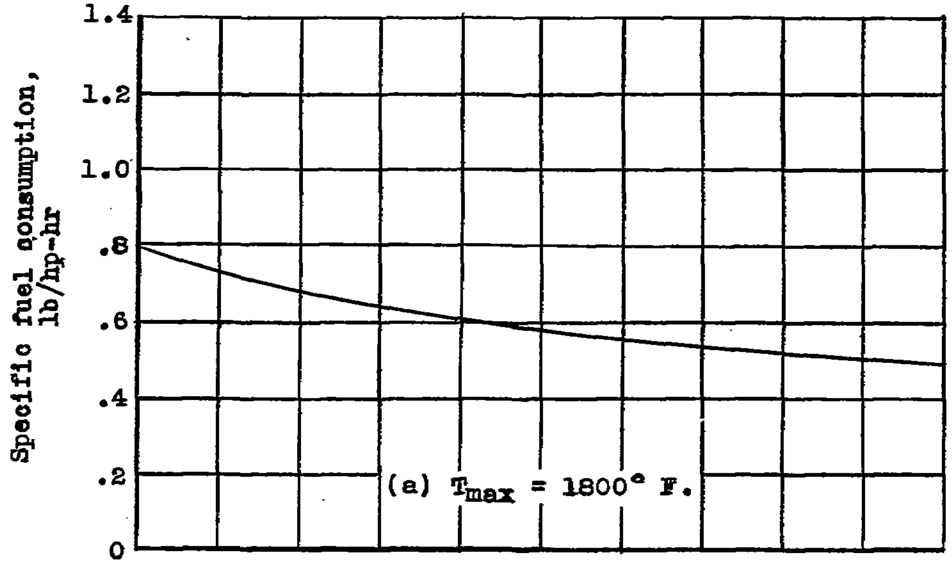
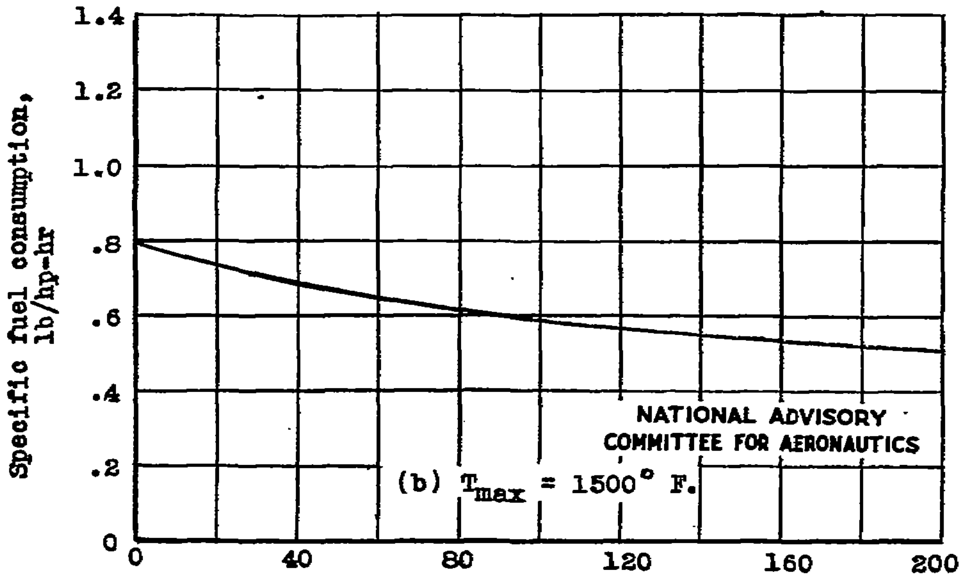


Figure 25.- Net change in weight of power plant and fuel per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; R_c = 4; A/W = 0.3 square foot per pound of air per second.





$\frac{\text{Regenerator weight}}{W}$, lb/lb per sec

Figure 26.- Variation of specific fuel consumption with heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; R_c = 3; A/W = 0.3 square foot per pound of air per second.

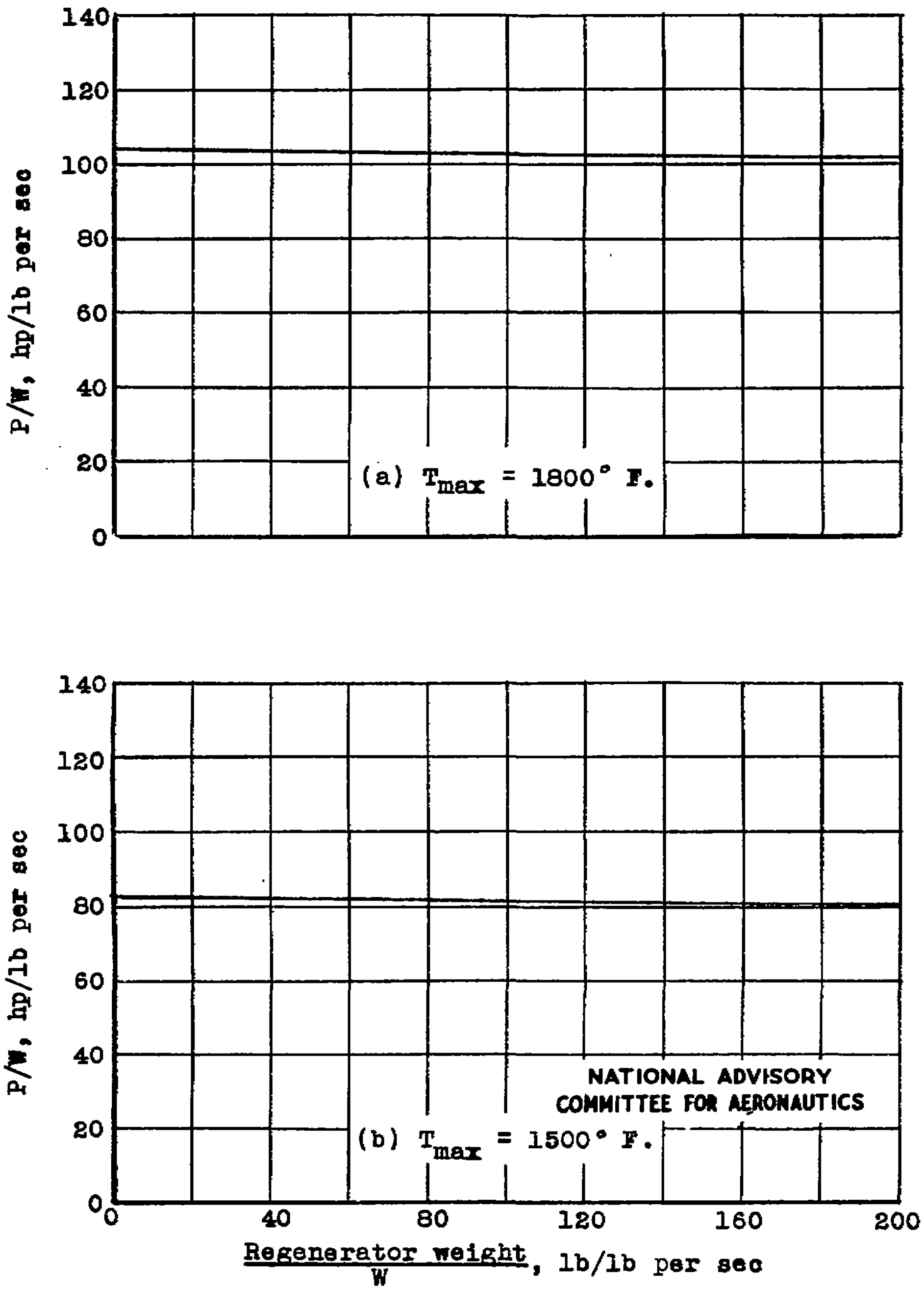


Figure 27.- Variation of thrust power per pound of air per second with heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; $R_c = 3$; $A/W = 0.3$ square foot per pound of air per second.

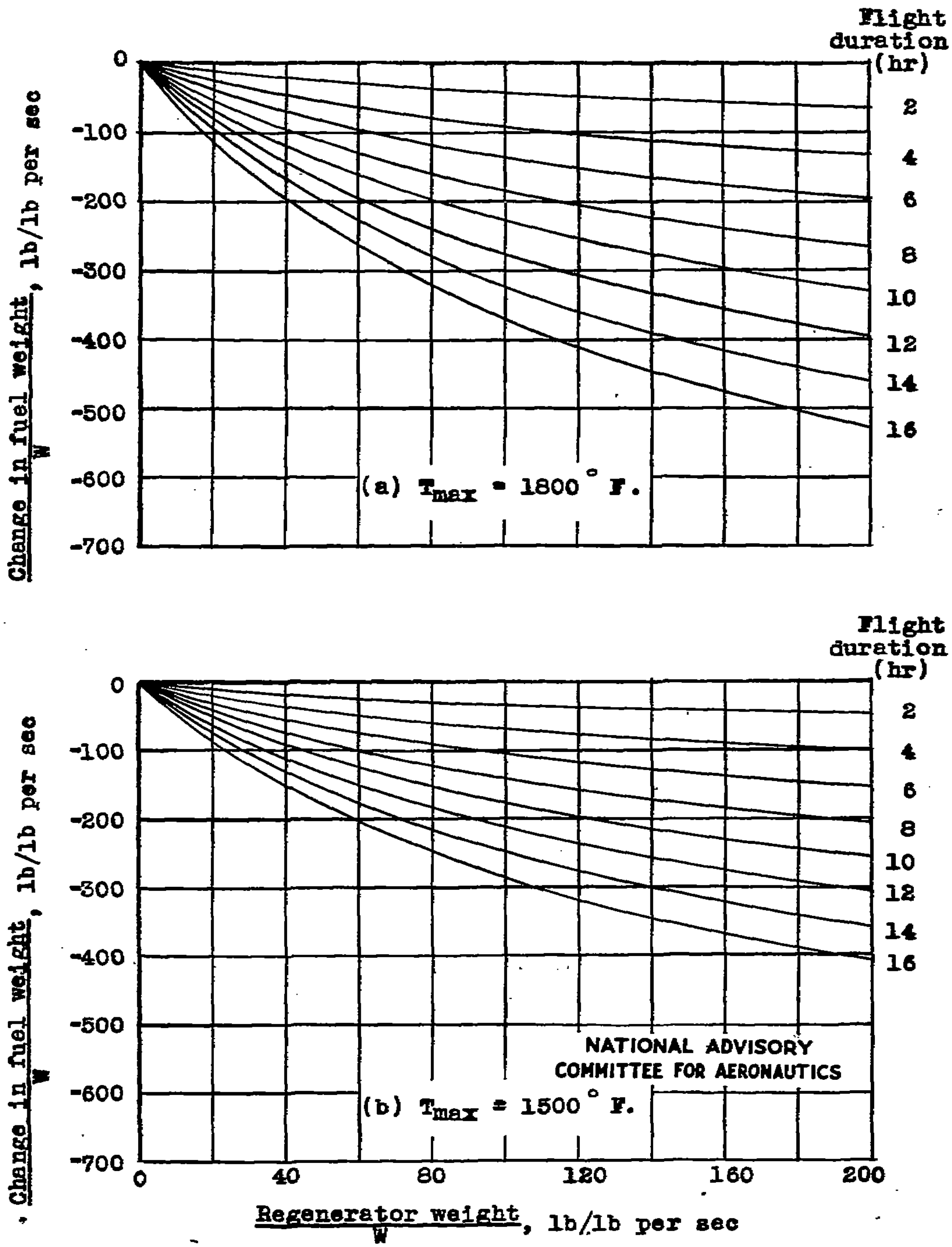


Figure 28.- Change in fuel weight per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; R_c = 3; A/W = 0.3 square foot per pound of air per second.

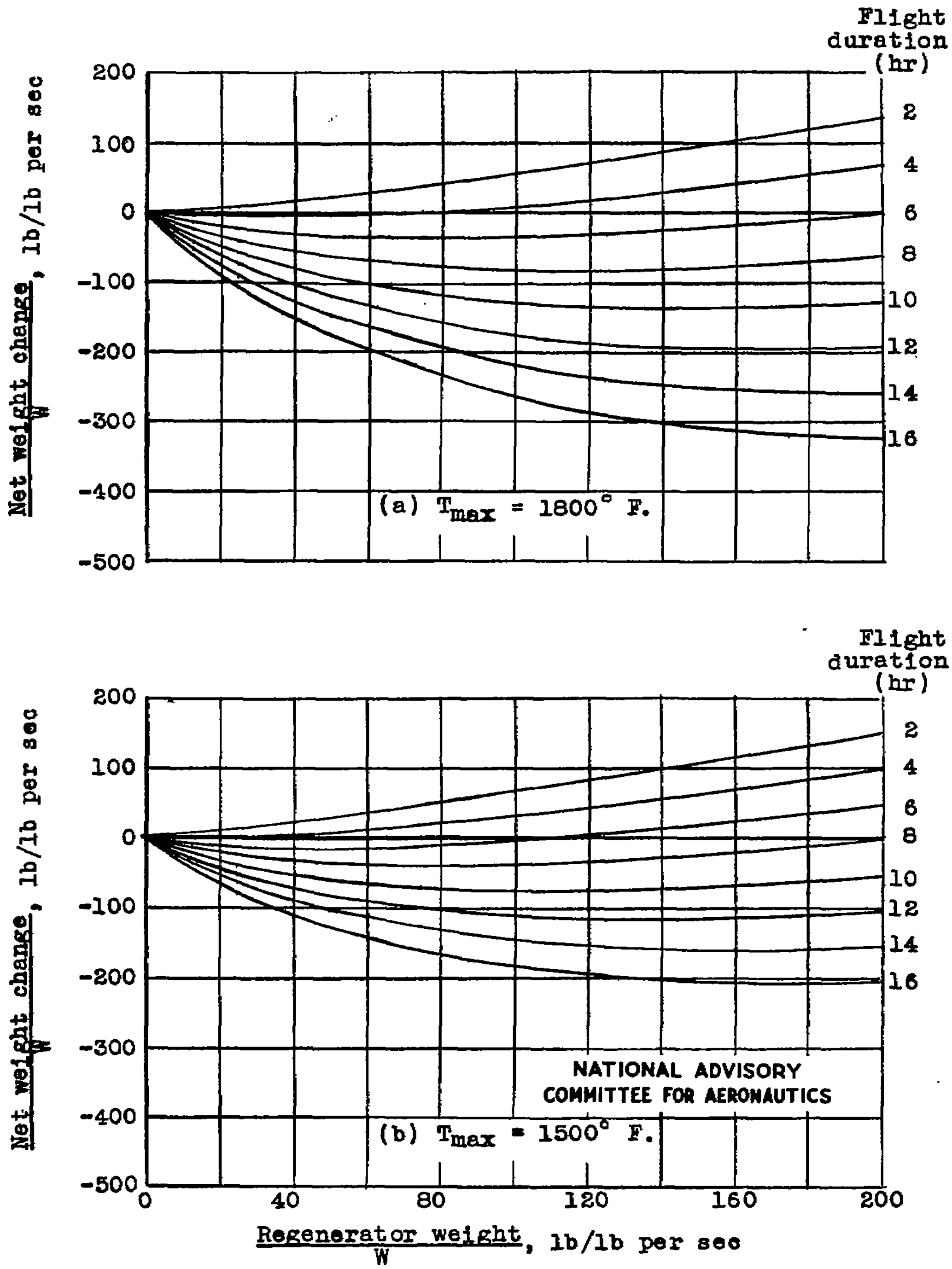


Figure 29.- Net change in weight of power plant and fuel per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; R_c = 3; A/W = 0.3 square foot per pound of air per second.

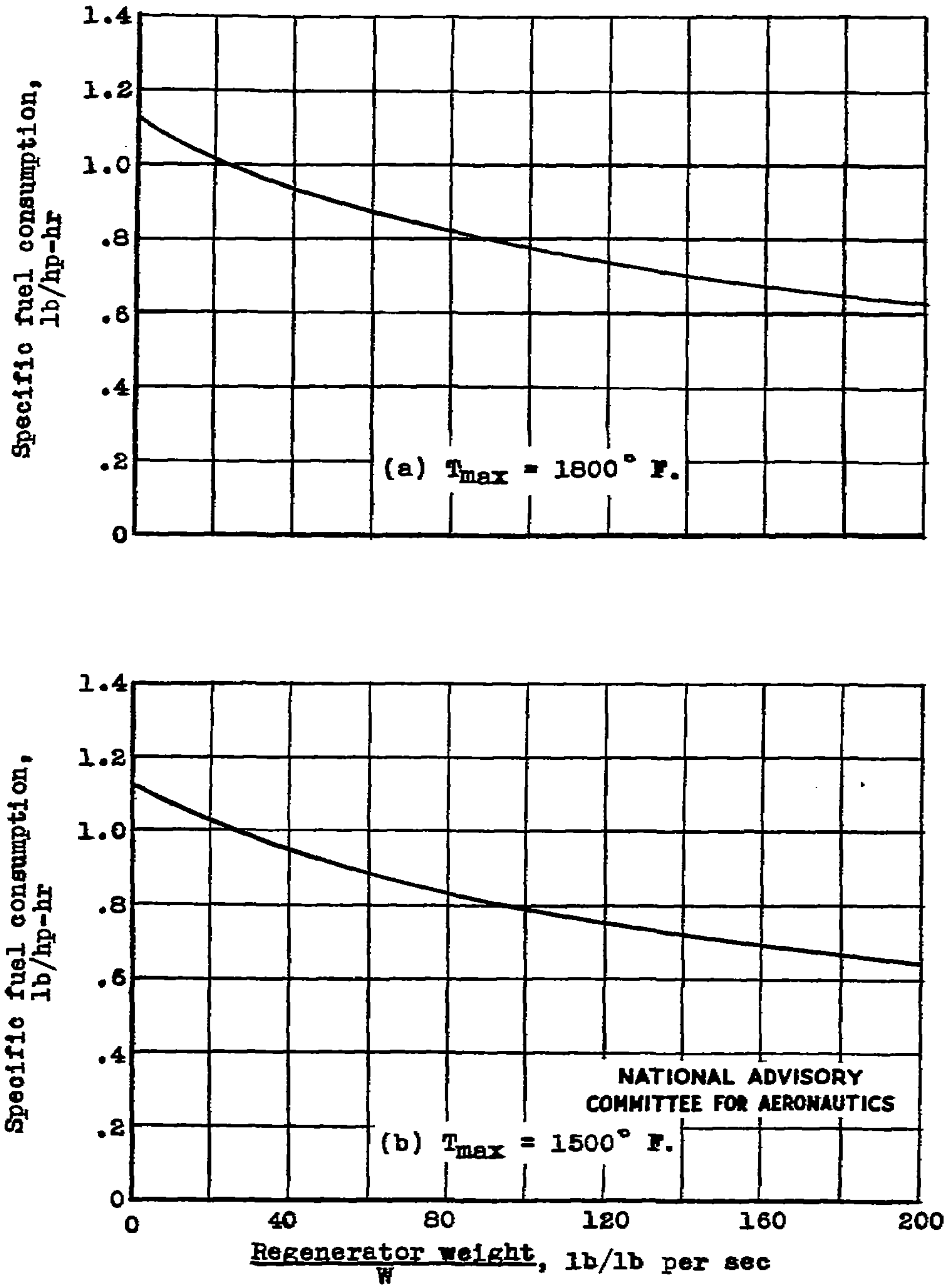
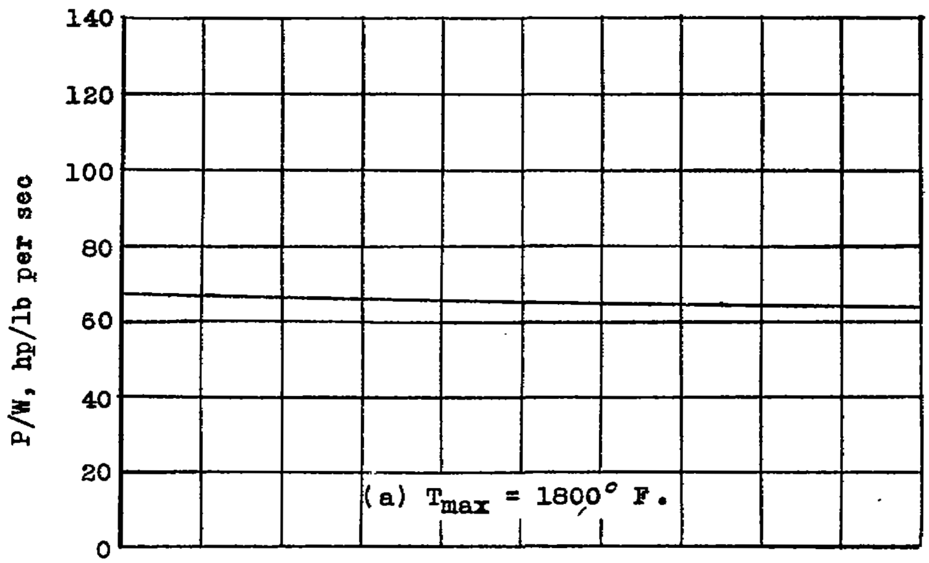


Figure 30.- Variation of specific fuel consumption with heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; R_c = 2; A/W = 0.3 square foot per pound of air per second.



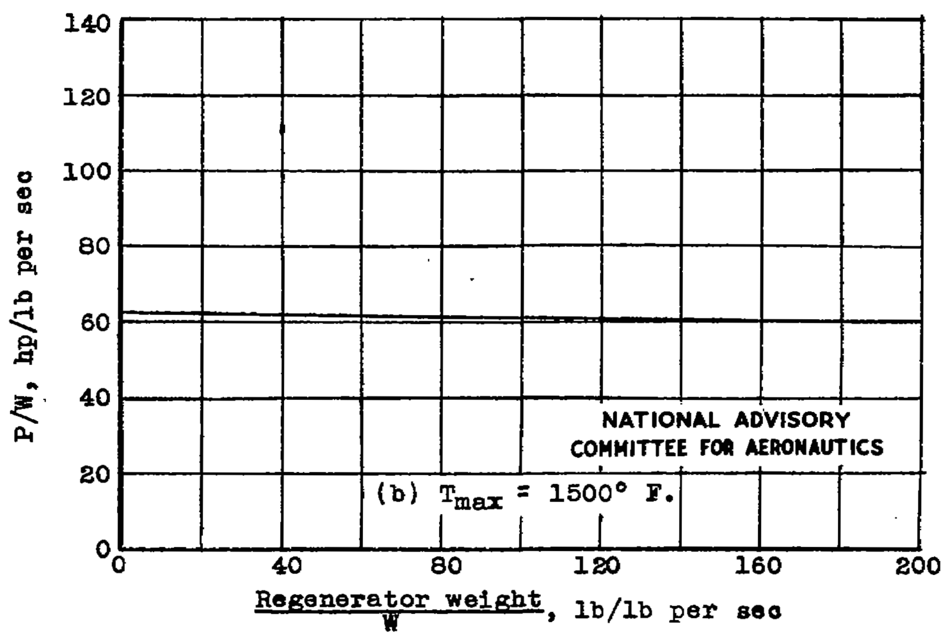


Figure 31.- Variation of thrust power per pound of air per second with heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; R_c = 2; A/W = 0.3 square foot per pound of air per second.

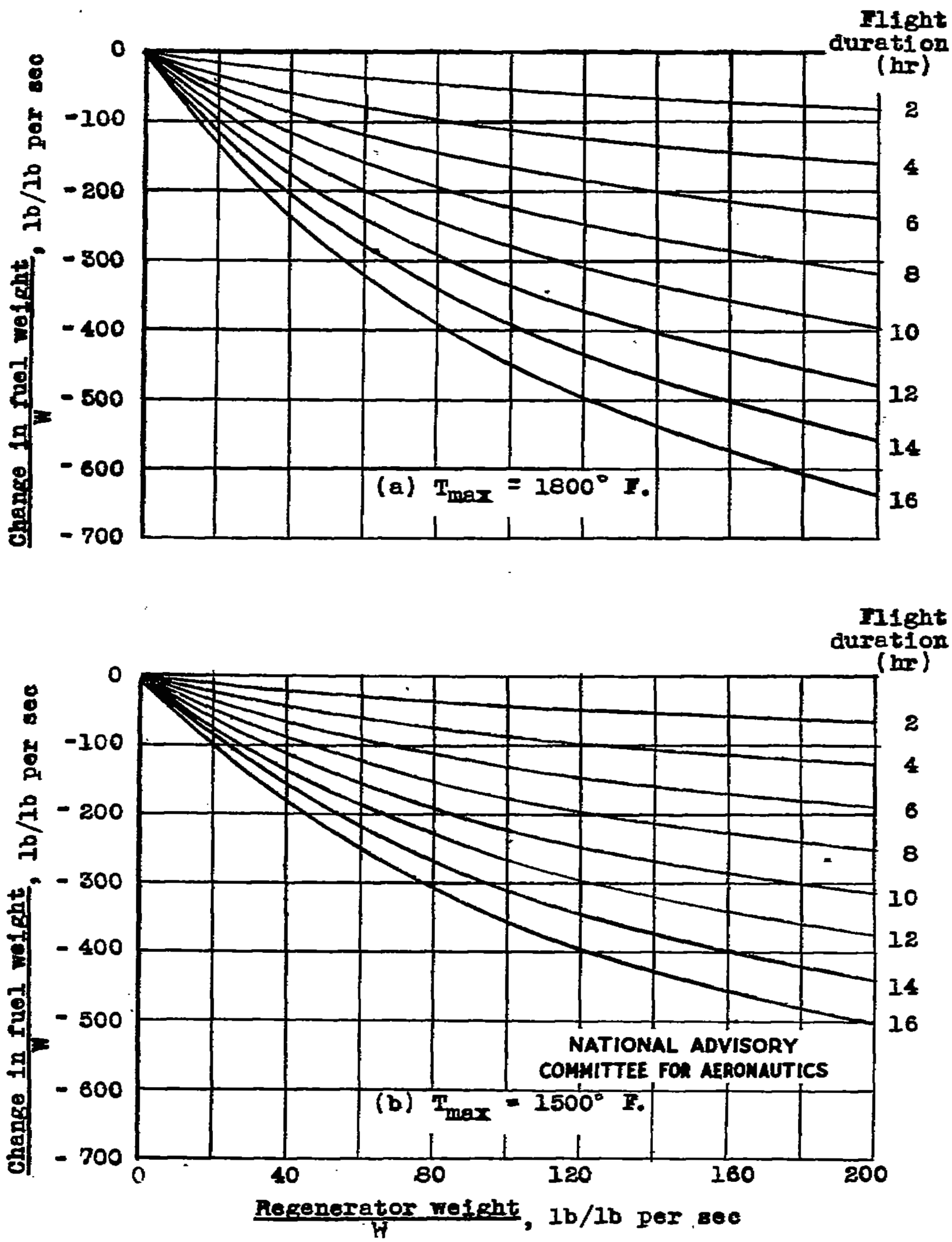


Figure 32.- Change in fuel weight per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; R_c = 2; A/W = 0.3 square foot per pound of air per second.

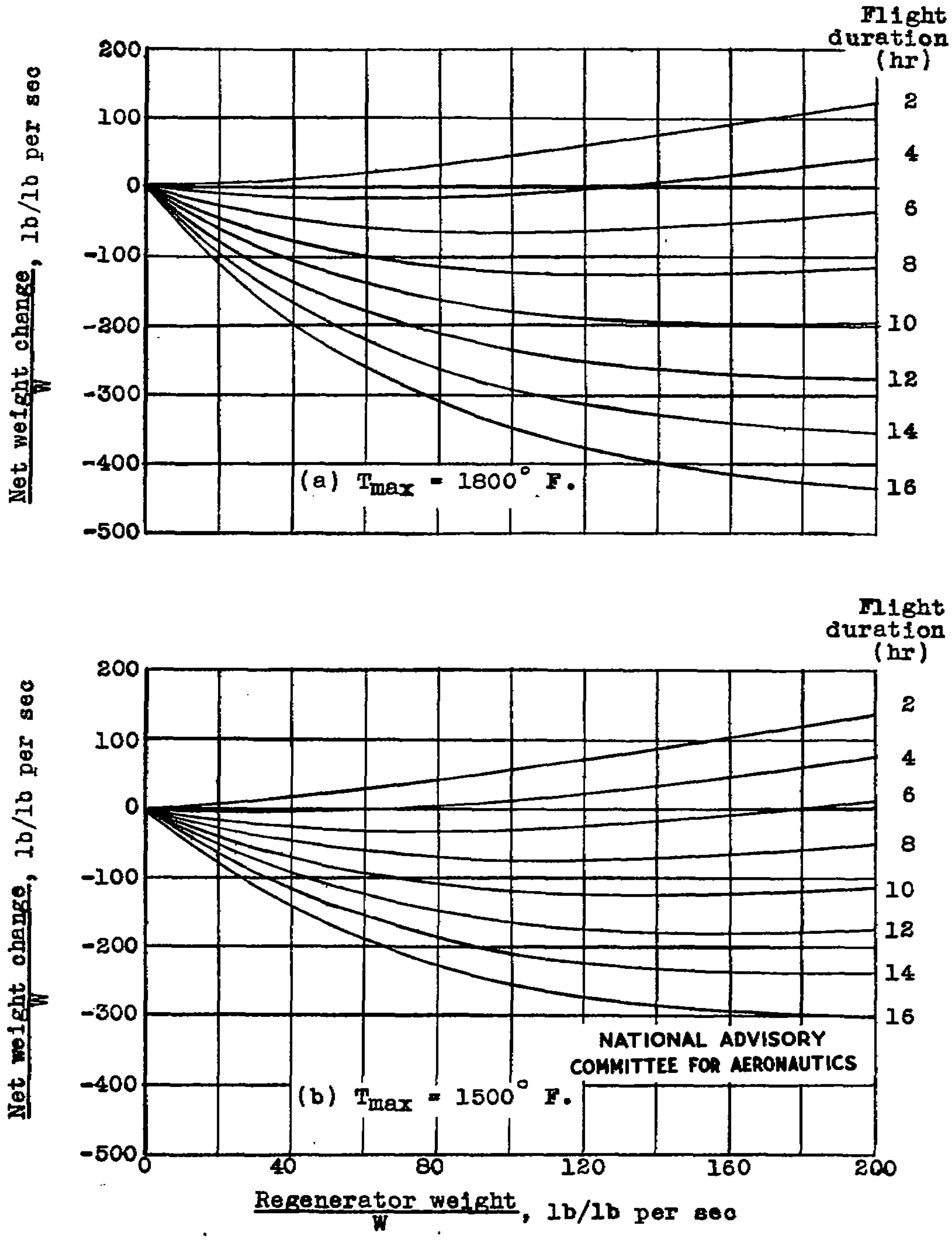


Figure 33.- Net change in weight of power plant and fuel per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, 25,000 feet; R_c = 2; A/W = 0.3 square foot per pound of air per second.

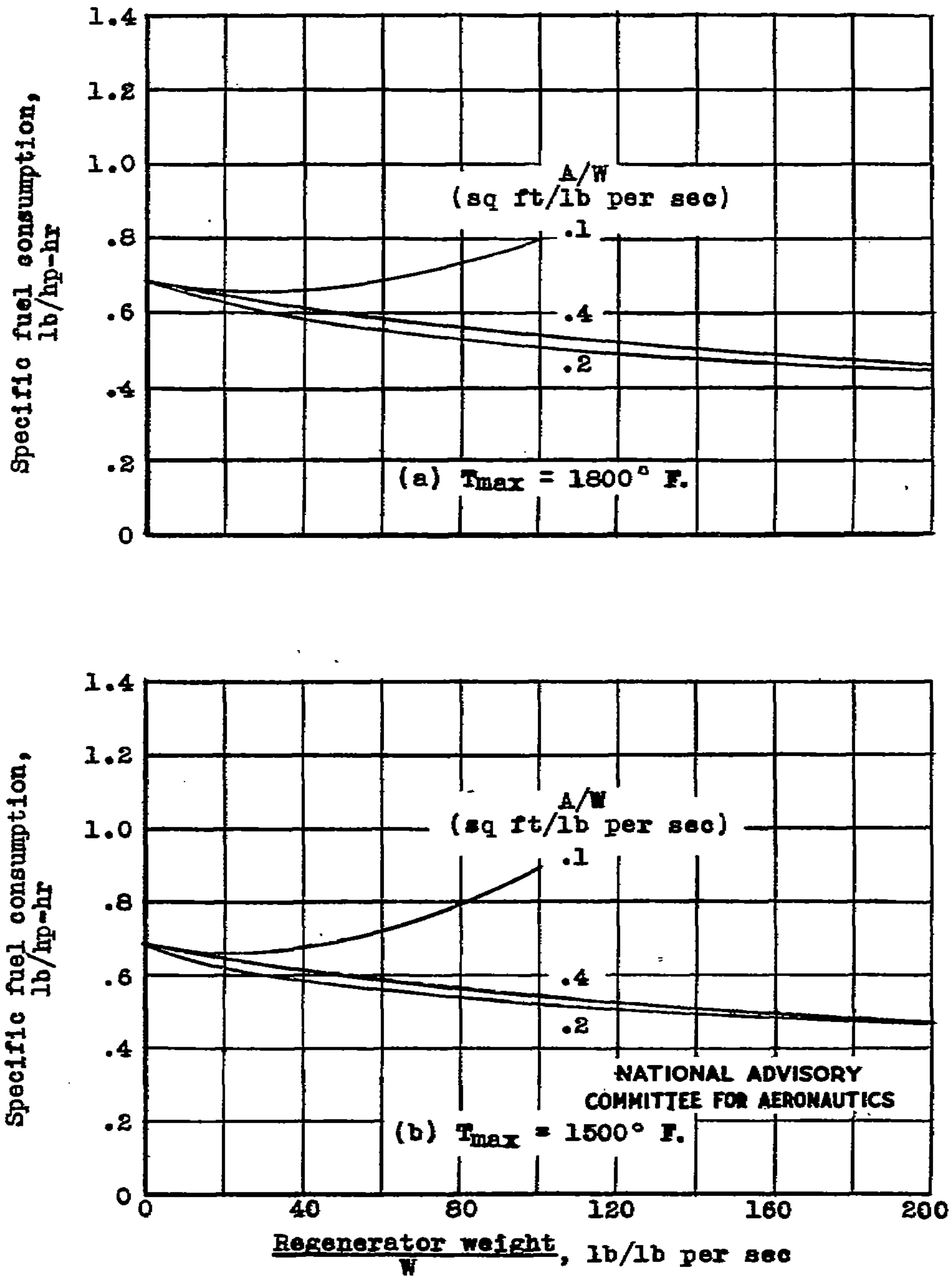


Figure 34.- Variation of specific fuel consumption with heat-exchanger weight per pound of air per second. Altitude, 40,000 feet; R_c = 3.

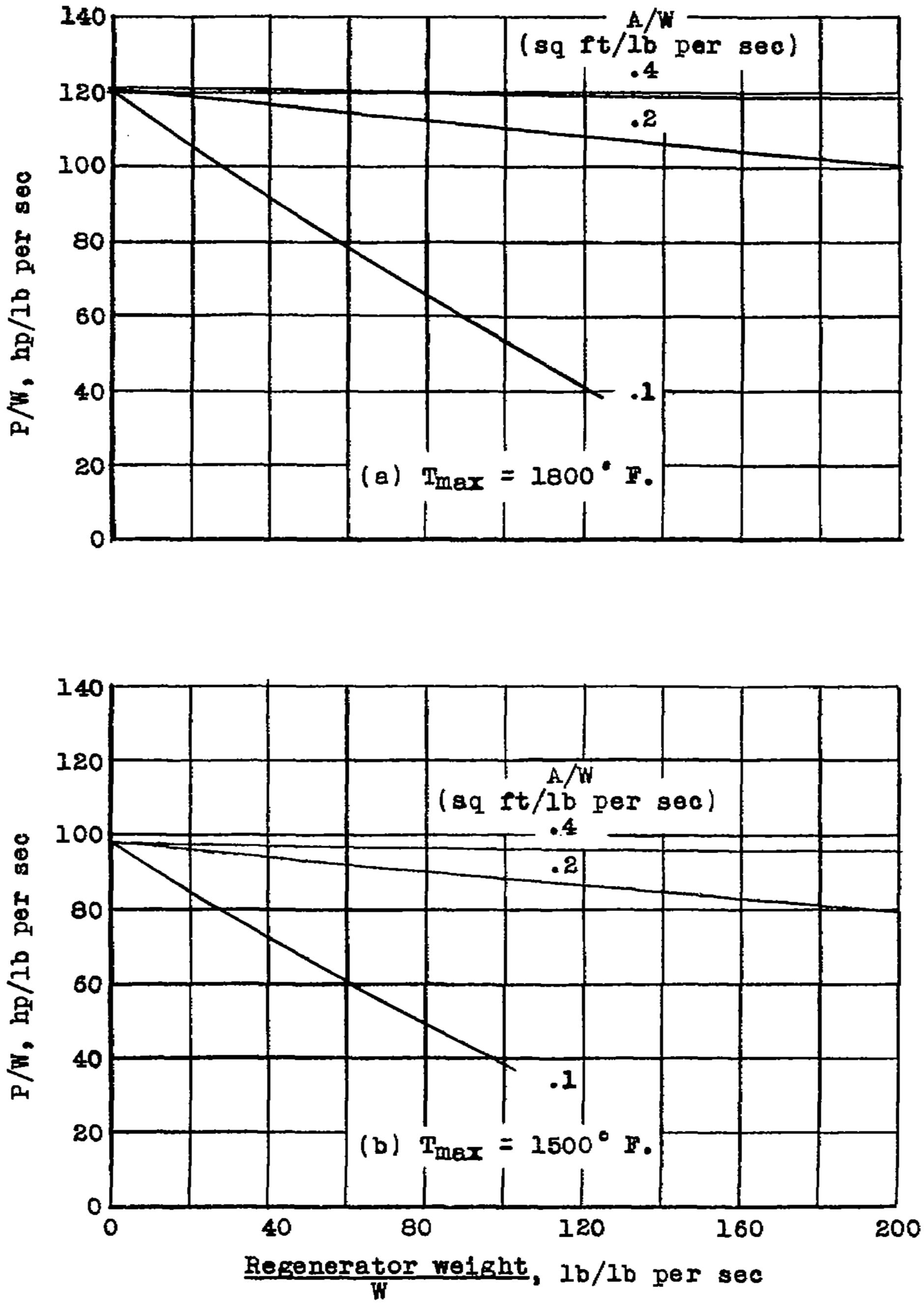


Figure 35.- Variation of thrust power per pound of air per second with heat-exchanger weight per pound of air per second. Altitude, 40,000 feet; R_c = 3.

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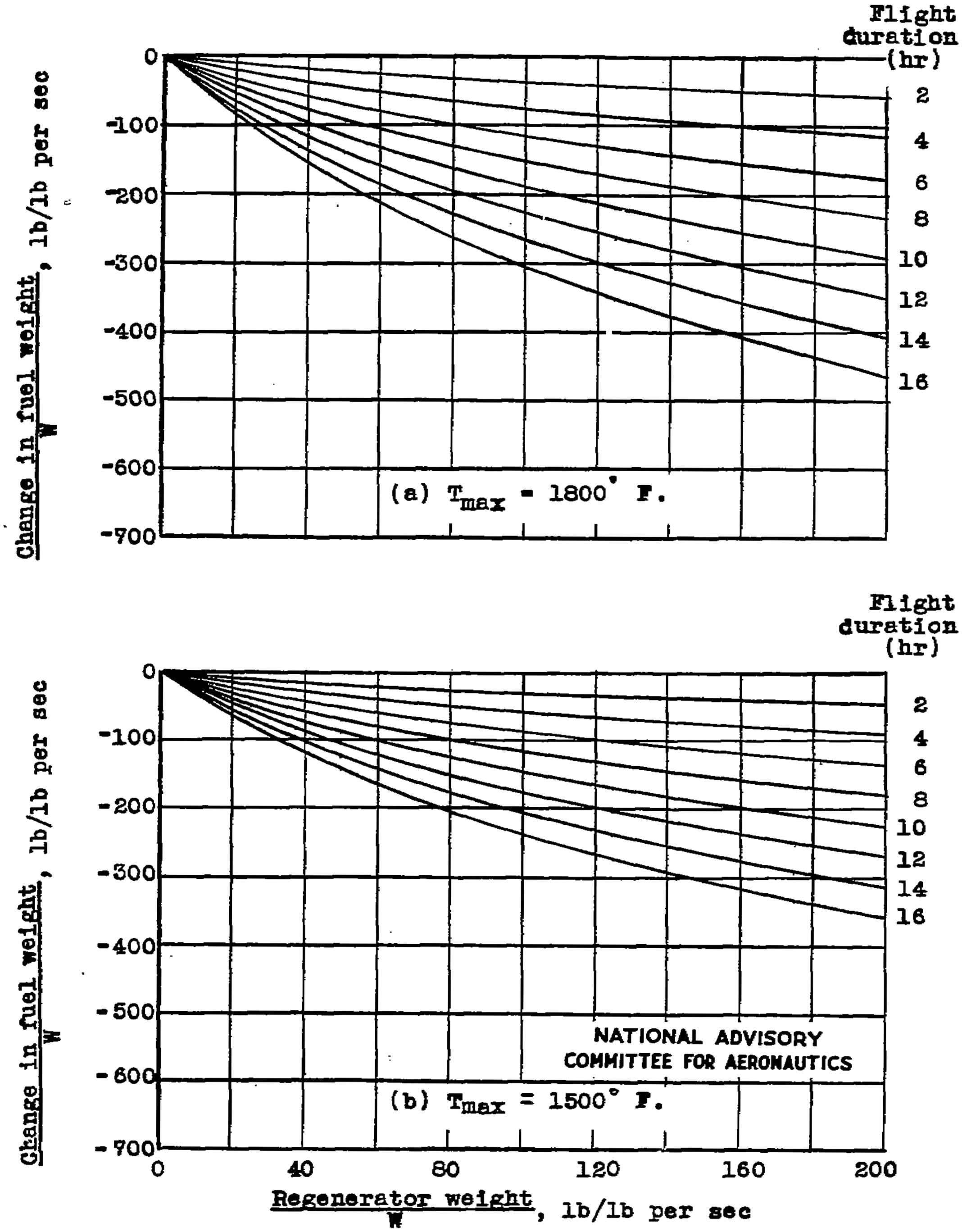

Figure 36.- Change in fuel weight per pouhd of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, 40,000 feet; R_c = 3; A/W = 0.4 square foot per pound of air per second.

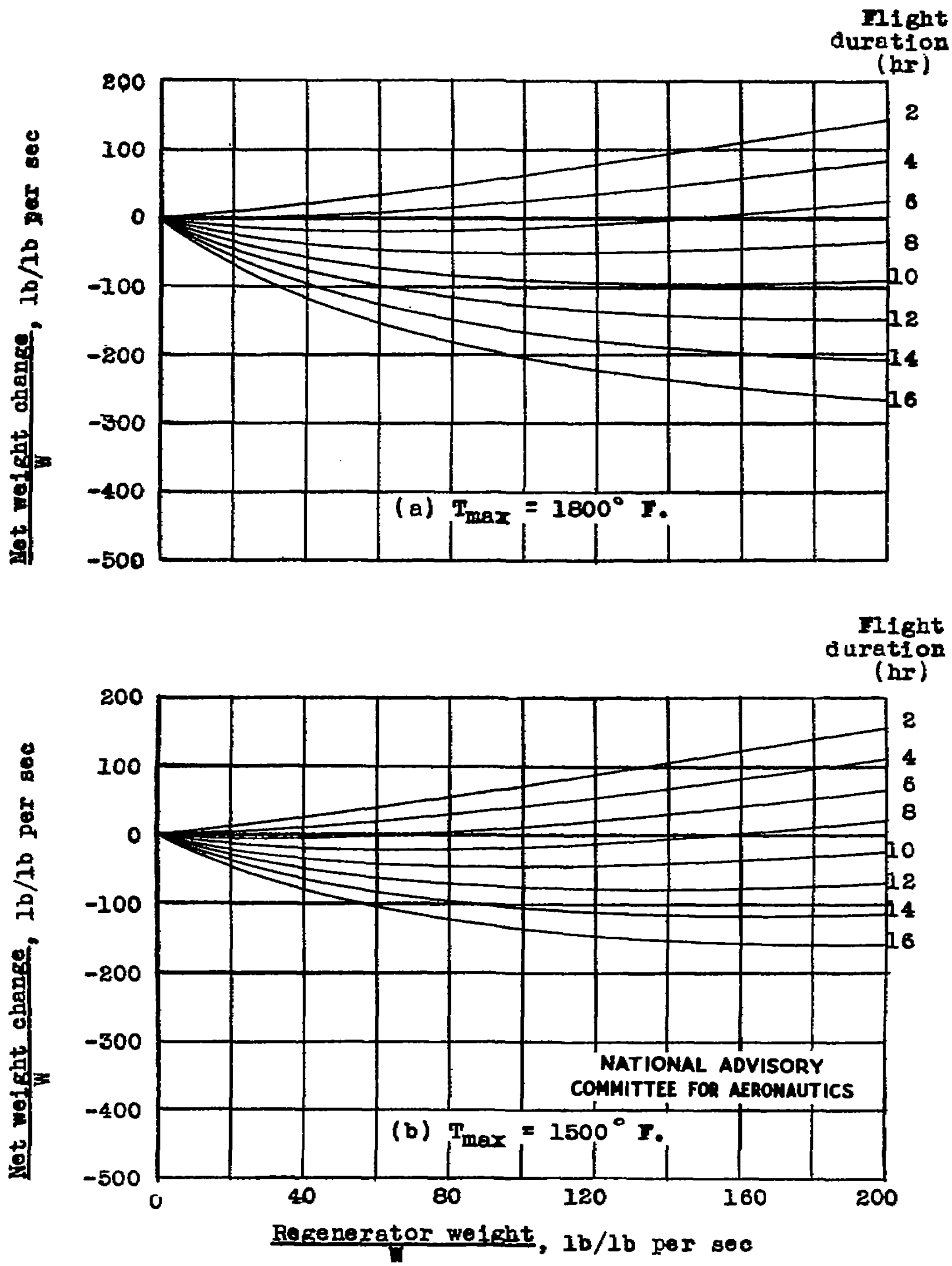

Figure 37.- Net change in weight of power plant and fuel per pound of air per second as a function of heat-exchanger weight per pound of air per second. Altitude, 40,000 feet; R_c = 3; A/W = 0.4 square foot per pound of air per second.

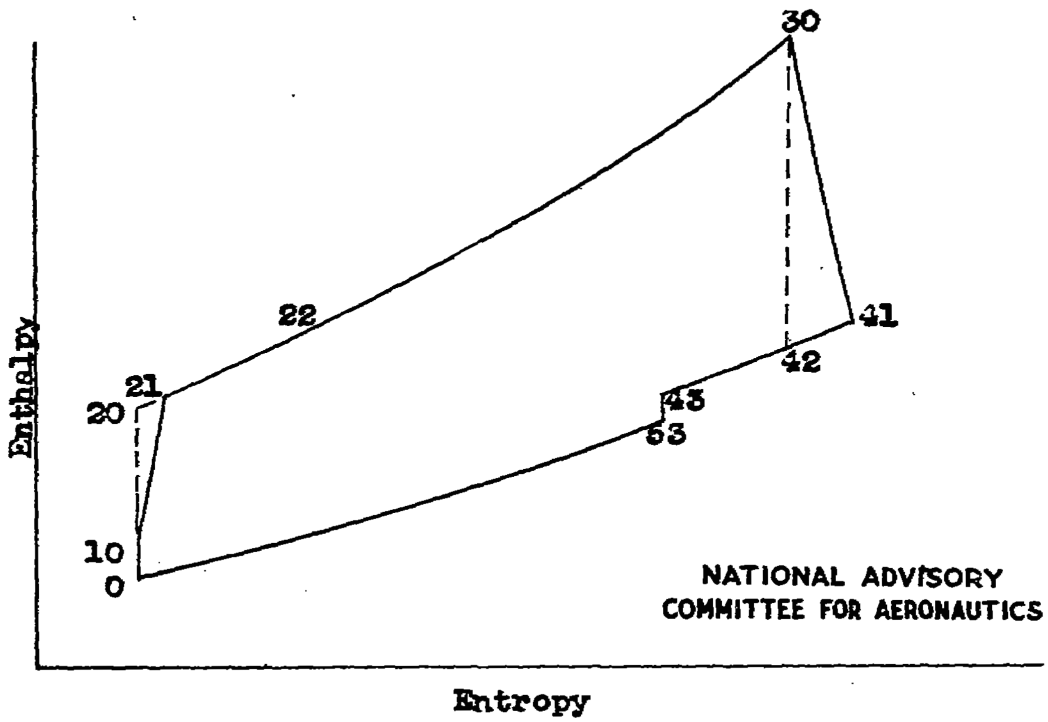

Figure 38. - Qualitative representation of enthalpy and entropy changes in regenerative cycle.

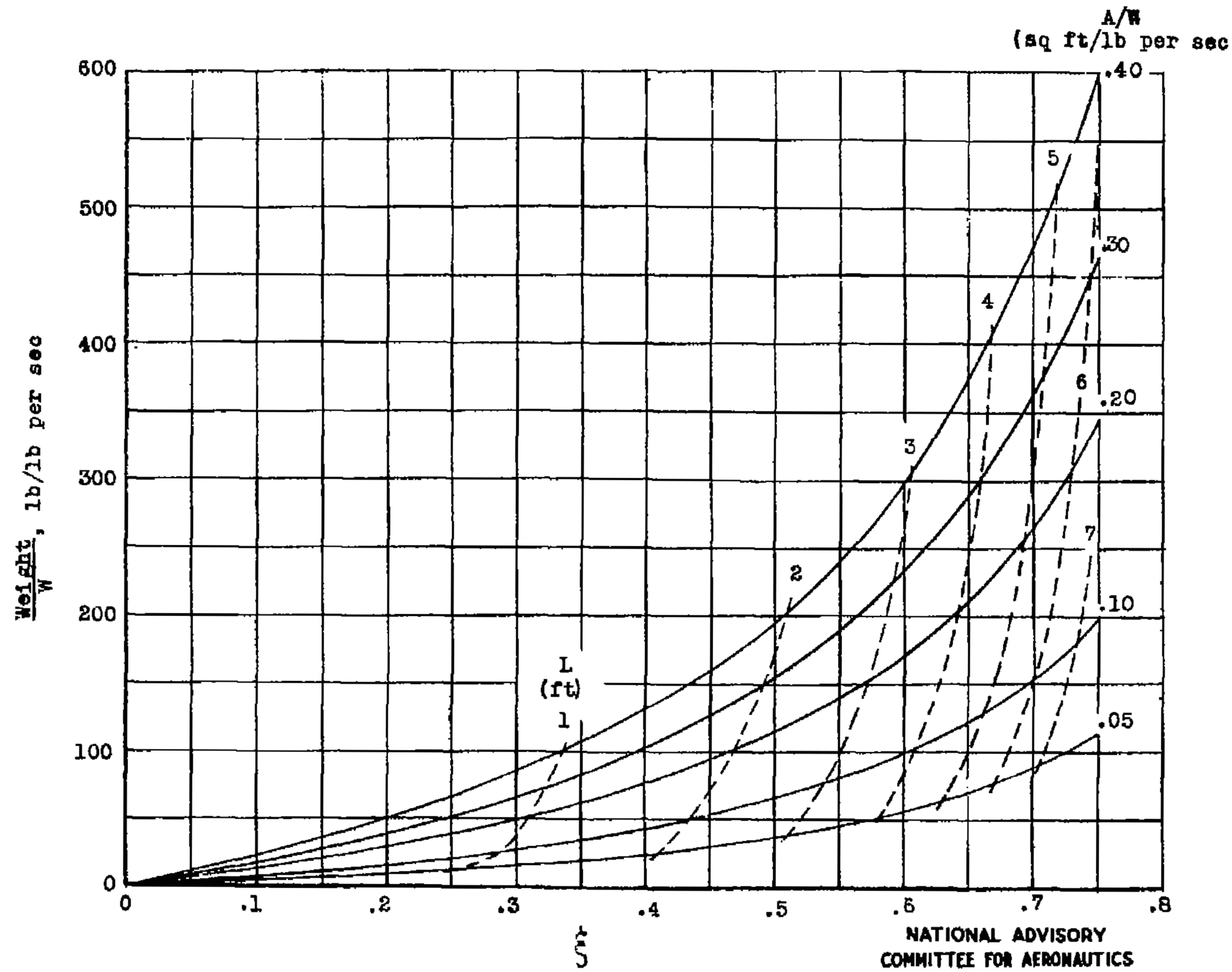

Figure 39.- Heat-exchanger weight and dimensions as functions of effectiveness.